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Use of Surface Acoustic Waves in Whispering Gallery Resonators for a Pan-Species Chemical Detector

David Covell, Matthew Letarte
Photonics Research Center
United States Military Academy
West Point, New York 10997, USA

Faculty Advisor: John Zehnpfenning

Abstract

The method of using opto-mechanical interactions to detect and predict the concentration or type of a fluidic pollutant in the environment surrounding a detector presents an empirical equation that predicts the type or concentration of fluidic pollutants via frequency shift. The model matches experimental results within 2%. Our device is a pan-species, highly sensitive pollutant detector. Current methodologies for detecting air pollutants require specialized equipment that is sensitive to particular compounds. A pan-species detector would be a useful and potentially cost effective method of sensing contaminants in an environment where the pollutants that may be introduced are not known a priori. It is possible to detect the presence of pollutants using an opto-mechanical Whispering Gallery Resonator (WGR). Opto-mechanical interaction occurs as laser light is evanescently coupled from a nearby fiber optic cable onto a resonator of circular geometry. Via photon-phonon interplay, a mechanical (phononic) Surface Acoustic Wave (SAW) and a red-shifted optical Stokes line are formed. The optical quality factor of such WGRs is on the order of half a billion and the acoustic quality factor is on the order of twelve thousand; such ultra-high quality factors lead to strong resonances, thus large mechanical deformation of the WGRs surface. The optical Stokes line couples back into the fiber and beats against the pump line at an opto-electrical detector. This beat note can be interpreted as the frequency of the SAW, via energy and momentum conservation. Through conservation of momentum we are able to determine the SAW velocity and therefore effective density of the fluid (gaseous or liquid) around the WGR. Presently, opto-mechanical interactions are predicted via simulation models using the Finite Element Method (FEM), which takes a long time to calculate for each iteration of data inputs. This is excellent for the lab, but impractical as an application where detecting harmful fluids in the environment quickly is essential. In order to be useful as a detector, we require an approximation to the data that can output results more quickly. Empirical study allowed us to develop an equation that takes into account the density of the fluid surrounding the WGR so that, as density changes due to introduction of pollutants, so does the predicted SAW velocity. Our research is to develop the model more fully so that, after further experimental results and improved computer modeling, we can predict substances that are introduced into the environment. Presently, experiments are under way to show detection of a variety of gaseous pollutants. Comparisons of detection time and sensitivity of experiment versus models are made.

Keywords: opto-mechanics, surface acoustic waves, whispering gallery

1. Introduction

Current methodologies for detecting the presence of harmful chemicals in the air rely on very limited means. The Department of Homeland Security helps to maintain a guide that identifies 207 detection devices that are capable of detecting various chemicals.[1] While M8 and M9 detectors, which use detection paper to determine the presence of chemical agents, are used most commonly by the military, there are other fieldable technologies that use spectroscopy or other means to identify whether or not harmful chemical substances are present in the air.[1] While these methods may be effective, they still require a careful balance between sensitivity and usability. That is because as a detector is made more sensitive, it is more likely that normal atmospheric pollution will set it off with a false positive.[1] Additionally, in order to ensure that a detector has accurately detected the presence of a harmful substance, it is often necessary to have redundant systems to verify readings. There is much room for improvement upon current detection techniques. We propose the use of opto-mechanical micro whispering gallery resonators (WGR).

In recent years, there has been a growth in the field of work relating to WGRs and forward Brillouin scattering. The examination of forward scattering is useful because it allows for the detection and interpretation of the scattered energy, something which would not be possible if the energy scattered backwards towards the source.[2-5] Of particular interest is the work done on electrostrictive actuational forces in WGRs like that seen in Figure 1. It was postulated that use of spherical WGRs would make it possible to measure the presence of chemicals in a fluid medium.[6] Expanding upon this work, it is proposed that a WGR would be capable of detecting a wide range of chemicals with a greater degree of accuracy than present detection techniques.

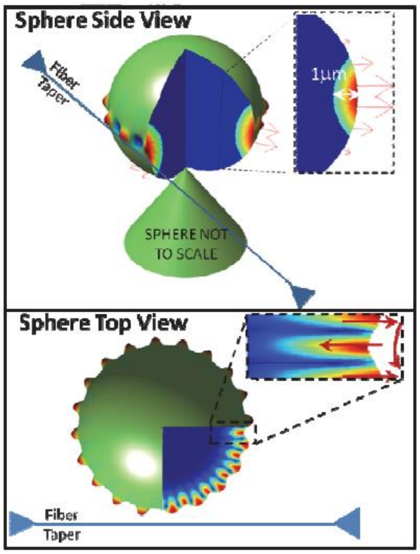

Figure 1. A cutout view of the electrostrictive force acting on a silica sphere. The SAW produces areas of high and low density which results in deformation as modeled by the maxima in the figure.[7, 8]

2. Theory

The WGRs function as chemical detectors via conservation of momentum. When a solid material interacts with a pump line, it is possible for an interaction to take place in which the photons collide with phonons and excite an acoustic wave that propagates along the surface of the solid. As the surface acoustic wave (SAW) begins to form, the pump line interacts with it and scatters a red-shifted laser line. This red shifted laser line is known as the Stokes line. The sum of the energy of the Stokes and acoustic modes equals that of the pump line. In this way, Stokes lines balance the conservation of momentum offset by the pump line's photon-phonic interaction and generation of the acoustic line.[8, 9] If the SAW has a momentum greater than that of the pump, it will settle at a k-vector twice the pump line and allow for the rise of backwards scattered Stokes line, while a SAW with momentum less than that of the pump line has a k-vector that varies with other variables and corresponds to a forward scattered Stokes line.[8, 9]

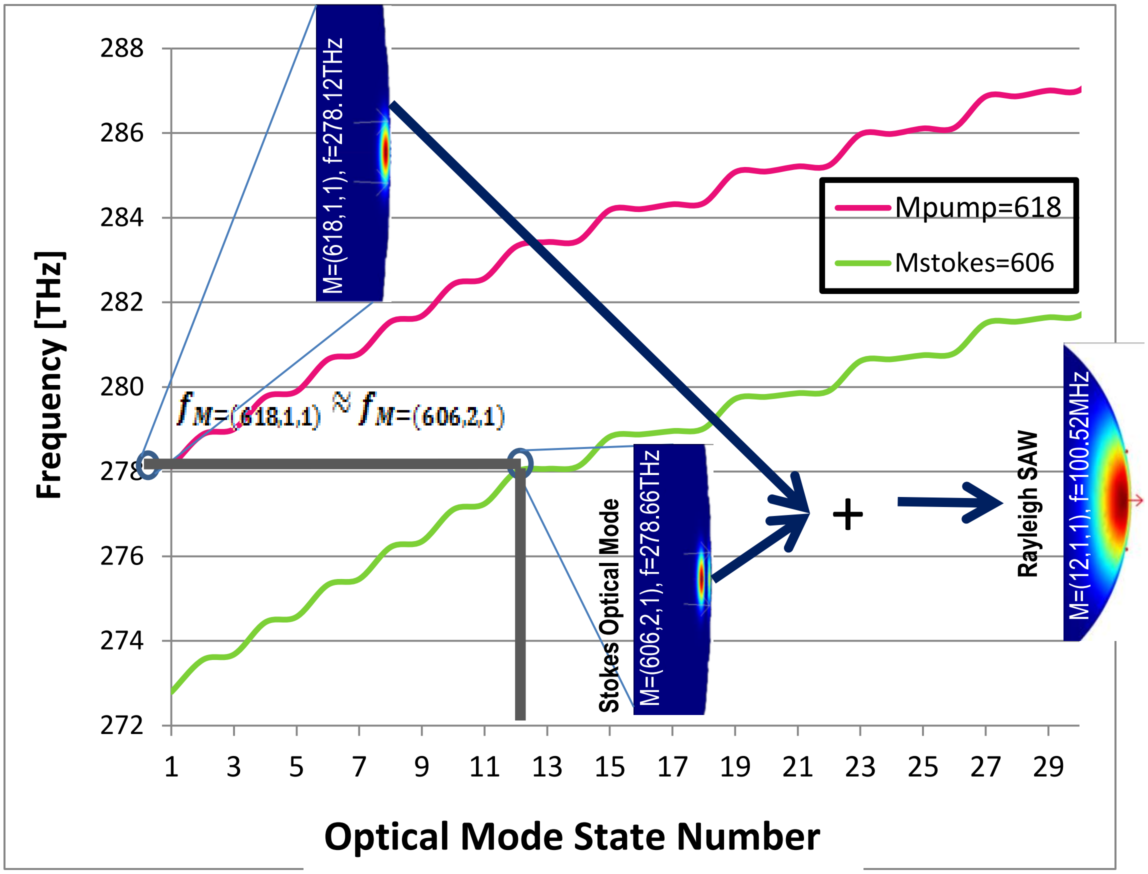

Figure 2. The pump line and the Stokes line both run out to the detection system and can be measured and characterized. The difference between the two incoming lines in energy and modal states by conservation of momentum is to be found in the SAW.

The Stokes and pump lines can be compared after passing into an Optical Spectrum Analyzer (OSA) or Electrical Spectrum Analyzer (ESA).[10] At a particular pump line frequency, a corresponding Stokes line is produced which has an optical mode number that is some number less than that of the pump line. The difference between the two modal numbers at a particular pump frequency is therefore, by conservation of momentum, to be found in the SAW wave traveling in the WGR. This relationship is represented in Figure 2.

The final velocity of the SAW is ultimately affected by the geometry of the WGR. Traveling through a planar surface, there are clearly established equations for determining the SAW velocity. These equations can be seen below in Table 1. However, the WGR is not planar, but a sphere with a radius on the order of microns. As a result of such a small radius, the SAW must constantly change direction and begins to protrude from the surface of the WGR. As it does so, the SAW interacts with the medium that surrounds the WGR and experiences changes in velocity.

Table 1. Analytic equations for various surface acoustic waves propagating through a planar surface and being affected solely by bulk properties.[7, 11-13]

Wave	**Velocity [m/s]**
Longitudinal	$V_L = \left(\frac{E(\nu-1)}{\rho(2\nu^2+\nu-1)}\right)^{1/2}$
Transverse	$V_T = \left(\frac{E}{2\rho(\nu+1)}\right)^{1/2}$
Rayleigh	$V_R = \frac{V_T(0.87+1.12\nu)}{(1+\nu)}$

After determining that the SAW velocity was affected by interactions with the environment, the following equation was proposed[7, 14, 15]:

$$v_{\text{saw}} = \alpha v_{\text{bulk}} + \beta v_{\text{fluid}} \tag{1}$$

This equation predicts the velocity of the SAW as a function of both the velocity of sound in the bulk material and the velocity of sound in the fluid that surrounds the WGR. The α and β multipliers are scalars that change the relative magnitude of the contribution of bulk or fluid velocities to the actual SAW velocity based on the geometry of the sphere.

Previously, in order to predict the SAW velocity for different azimuthal mode numbers, the Finite Element Method (FEM) was used[16-19]. Azimuthal mode numbers correspond to different radii because the sphere has to have a circumference that is an integer multiple of the SAW wavelength. This is necessary so that constructive interference can take place and the SAW propagate. The results of FEM calculations enabled us to produce Figure3, which shows high SAW velocities at low mode numbers and which decayed at higher mode numbers with an asymptote at the value given for a planar SAW wave.

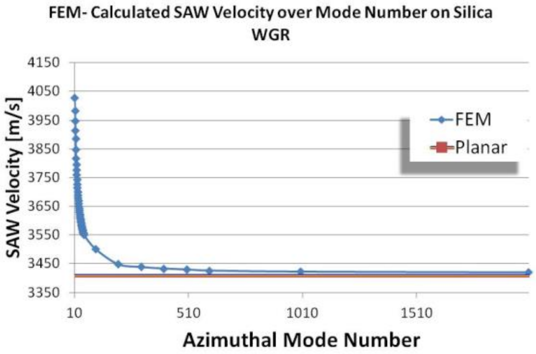

Figure 3. Using a Rayleigh wave approximation and comparing it with the results of the FEM model, it is seen that the FEM converges with the planar wave approximation at large mode numbers as expected.

Although the FEM provides a solution for SAW velocity, there is one major shortcoming that instigates the search for a better mathematical model of the SAW to environment relationship. That shortcoming is time. The FEM takes several minutes to process data for a single iteration. In a system that will be constantly processing data and needs to have results in a matter of seconds, a better approximation needs to be developed.

3. Experimental Method

This experiment begins with taking a telecom wavelength tunable laser and coupling it into an SMF-28 fiber. The light passes through the fiber and is confined to a fiber waveguide that has been mechanically tapered such that its entire radius is equal to a diameter that supports a single mode of the pump line.[20] At that point an evanescent light wave passes out into the environment and interacts with the WGR, causing a photonic-phononic interaction on the surface.[9] This interaction begins the production of a SAW along the circumference. The light power from the evanescent waves also passes into the WGR and begins to circulate along the circumference of the sphere, amplifying through constructive interference on every circulation.

Once the Stokes line is established and amplified, it gains enough power to evanescently couple out of the WGR and into the fiber core and continue to the measuring equipment. Passing the output of the fiber into an OSA or ESA, it is possible to see the pump line of the laser beat against the Stokes line to create a beat note. This sets a baseline frequency for the beat note. It then becomes possible to observe any variations in the frequency and to interpret whether or not a harmful pollutant has been sensed in the atmosphere. The sensitivity of this system is currently set by the pump line width of 10Hz. Beat note shifts of fewer than 10Hz cannot be detected with certainty due to the fact that line movement can be attributed to random noise. However, shifts of greater than 10Hz can be detected and help to signal that there has been a SAW frequency change.

4. Results

We captured data and evaluated the SAW velocity at different azimuthal mode numbers. In order to determine the exact radius of the WGR, a high powered microscope was used, and a scale determined the actual radius of the sphere. After taking 6 data points, it was possible to see that there was a curve fit that could connect the data points. This curve fit also came very close to that of the FEM approximation that was run, helping to validate the FEM approximation as closely describing actual results with about 3% error. The results are in Figure 4. Although the FEM results are excellent approximations of the experimental excitation, the time for a model to converge and then sift through the results is not practical for detecting and identifying environmental changes rapidly. A faster but equally accurate approximation is desirable.

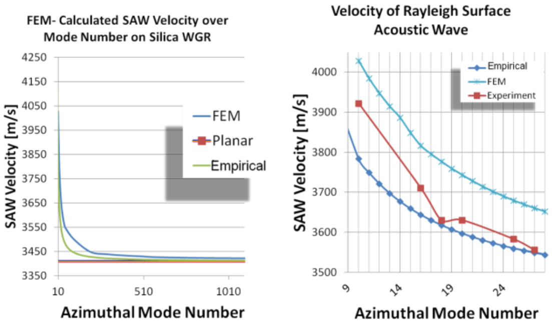

Figure 4. Comparison of FEM-approximated, empirical equation approximates, and experimentally found surface acoustic mode velocity versus mode number. Left: results over all predicted forward- and backward- scattered modes. The non-sloping line at 3410[m/s] is the predicted planar velocity in silica. Right: zoomed-in chart showing prime forward-scatted modes, method comparison. In both charts one notes the close approximation between the calculations and experimental results.

After empirical analysis, the values to α and β were determined. If we define $\zeta = \frac{\lambda}{r}$ where λ is the wavelength of the pump laser and r is the radius of the WGR, the equations for α and β are[14]:

$$\alpha = (1 + \mathrm{Sin}[\zeta]) \tag{2}$$

$$\beta = \mathrm{Sin}[\zeta] \tag{3}$$

Placing these values into the equation for v_{saw} gives:

$$v_{\text{saw}} = (1 + \text{Sin}[\zeta])v_{\text{bulk}} + \text{Sin}[\zeta]v_{\text{fluid}} \tag{4}$$

This equation very clearly illustrates how the relative strengths of the two velocities stems from the measure of the radius as it compares to the wavelength of light, and that as radius increases the SAW behaves more and more as if it is on a planar surface.

There is a limit to how large the radius can get before the system begins to behave as a planar surface. According to equation 3, as $r \to \infty$, $\beta \to 0$. When $\beta = 0$, the only factor affecting the SAW velocity is the speed of sound in the bulk. Smaller-diameter WGRs have a larger apparent radius of curvature relative to the telecom-wavelength. This leads to a stronger interaction with the environment, thus smaller WGRs are preferable. The mechanism for this momentum conservation is a permutation of a phenomenon known as Brillouin scattering. Brillouin scattering theory shows that there is a limit on the possible azimuthal mode numbers of the surface wave, due to momentum conservation.[21] Thus, there is a segment of the modal spectrum, approximately between modes 50 and 200, where no acoustic modes are possible. Brillouin theory predicts that acoustic modes between 1 and 50 maxima are also not possible due to a requirement of an infinitely long acoustic wavelength. This limitation holds for Brillouin back scattering, however we have shown that such modes are possible if detected in a forward-scattered direction[7, 9, 19], which we refer to as a forward-scattered Brillouin process.

Using equation 4, it is possible to rewrite the velocity in terms of frequency and then solve for the SAW frequency. Since it is known that:

$$v_{\text{saw}} = f_{\text{saw}} * \lambda \tag{5}$$

we write equation 4 as:

$$f_{\text{saw}} = \frac{v_{\text{bulk}}}{\lambda} + \frac{(v_{\text{bulk}} + v_{\text{fluid}}) * \text{Sin}[\zeta]}{\lambda} \tag{6}$$

Using equation 6, it is possible to evaluate the relationship between concentrations of particular gases and the SAW frequency to which they correspond. A sample of some of these relationships at low concentration is shown below in Figure 5.

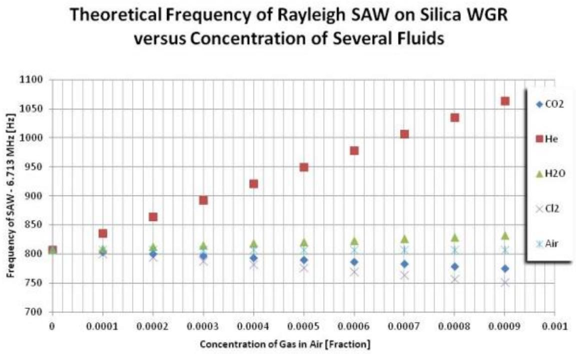

Figure 5. The current WGR detection system requires a 10 Hz shift for detection. Using known chemical and bulk properties it is possible to model probable effects of low chemical concentrations on SAW frequencies and then calculate a slope of the response. Each chemical seems to have its own characteristic slope response that could be used to help predict chemical pollutants.

The utility of this relationship can be considered using the example of chlorine gas. At a concentration of 400ppm, chlorine gas is lethal to a human after 30 minutes of exposure. According to calculations, at that concentration there is a shift from the SAW frequency in pure air of about 30Hz, which is well over the 10 Hz sensitivity of the WGR detector. If a program maintained vigilance of the SAW frequency and detected a steady shift, it could use Boyle's Law to determine how fast the frequency is changing relative to concentration and provide a slope. The slope of each chemical is unique, and so by maintaining a library of the slope of each chemical, a profile could be matched to the unknown chemical so that experts know how to deal with the threat.

5. Conclusion/Future Directions

As there are with all new technologies, there are limitations to the proposed method of detection. As it was explained here, this method does not have the ability to profile a gas that is introduced in high rapid concentration at the site of the detector. A frequency shift would alert the detector that there has been a sudden and likely dangerous, change in ambient conditions. However, the detector will not be able to determine with any degree of certainty whether mustard gas, chlorine gas, a nerve agent, or a passing steam engine is the cause of the shift.

Another possible issue for this detector comes when multiple gases enter the detector at the same time. The compound effect of these two gases would create a frequency shift that would appear to be that of a third chemical, one that is equivalent to their summed effects. This could result in a false positive for a harmless agent, when in fact two harmful agents were present. Or it could cause the incorrect safety measures to be taken.

However, it seems that with some work the use of a SAW in a WGR would be a feasible detection method that operates equally for all types of chemicals. Even should the detector prove unable to exactly pin down the chemical agent present, it would alert personnel that there has been a change in the environment and allow them to take certain precautions in advance of an actual diagnosis.

Additionally, it would be possible to profile the area of a detector prior to a gas attack in an effort to filter out typical background chemical or pollutant noise. This would allow the detector to be more accurate. Instead of testing positive due to the normal presence of smog, the device analyzing the detector's output would know that at certain times there are certain conditions that preclude pure air, and so a recalibration or setting of a minimum threshold would help prevent false positives from occurring.

Presently and into the future we continue to refine our models and gather experimental data. We expect that it is possible to develop libraries of known pollutants – both pure and mixtures. Important work continues and may lead to advances in chemical detection and identification.

6. Acknowledgements

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