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Experimental Progress on Tests of Gravity at 20 microns

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Abstract

Due to the incompatibility of the Standard Model and General Relativity, tests of gravity remain at the forefront of experimental physics research. At Humboldt State University, undergraduates and faculty are developing an experiment that will test gravitational interactions at the twenty-micron distance scale. The experiment will measure the twist of a torsion pendulum as an attractor mass is oscillated nearby in a parallel-plate configuration which will provide a time-varying torque on the pendulum. The size and distance dependence of the torque variation will provide means to determine deviations from accepted models of gravity on untested distance scales. To observe the twist of the pendulum inside the vacuum chamber an optical system with nano-radian resolution is required. This paper will provide a general overview of the experiment, as well as address the measurement and characterization of environmental systematic effects that must be understood in order to achieve the required sensitivity.

Keywords: Gravity, Inverse-square Law, Weak Equivalence Principle, General Relativity

1. Introduction

Since 2007, the Humboldt State University Gravitational Research Laboratory (HSU Gravity Lab) has been developing an experiment featuring a torsion pendulum to probe gravity at the sub-millimeter distance scale. The HSU Gravity Lab offers the only research opportunity for undergraduates in the Department of Physics and Astronomy. The lab was designed to produce tests of the Weak Equivalence Principle (WEP) of General Relativity (GR) and the gravitational inverse-square law (ISL) at unobserved distances scales. The WEP of GR states that the motion of a test mass under the influence of a gravitational field is independent of the composition of the object. Tests of the WEP have been conducted with incredible precision over distance scale from 1 cm to ∞ and, and within the error bars of the experiment, the results were found to be consistent with GR predictions. Investigations into the WEP at the sub-millimeter distance scales have not been conducted to date, and there are no other known efforts underway to do so. The ISL predicts that the force of gravity between two objects is dependent on the inverse-square of the distance between them. From previous experiments, the ISL has been found to be valid within experimental uncertainty from distances of ~50 μm to planetary scales. However, some models of Dark Energy, MONDS, and String Theory predict that the ISL might fail over galactic distance scales or at very small separations. If any deviations from the expected ISL or WEP behaviors were found, this would be an indication of unobserved properties of gravity or new physics. Understanding how gravity operates at the sub-millimeter scale might provide insight to how the Standard Model of particle physics and GR can unite.

2. Background and Motivation

Much of modern physics has looked to tests of gravity for insight into a unified theory that would combine the Standard Model and GR. General Relativity successfully describes gravity and has passed all experimental tests to date; however, it is inconsistent with the Standard Model of particle physics that is used to describe the other three fundamental forces. Additional spatial dimensions (apart from the known three) presented in String Theory allow for a potential unification of GR and quantum mechanics. However, a consequence of having these compact extra dimensions is that at the sub-millimeter distance scales there might be a measurable deviation from the ISL[1,2].

The WEP is a key feature of GR; therefore, any deviation from it at any length scale will call for a modification of the current theory of gravity or can be evidence of new forces mediated by exotic particles[3]. In addition, recent observations of the cosmic distance scale acceleration, attributed to Dark Energy, have enticed speculation that gravity may be fundamentally different from how it is now understood. One attempt to explain the cosmic distance scale acceleration is to have gravity "turn off" at distances less than about 0.1mm to be consistent with the astronomical data[4]. This particular prediction results in a violation of the ISL in the sub-millimeter regime.

It is traditional to model deviations from the ISL by including a Yukawa addition[4] to the classical Newtonian potential. For point masses m_1 and m_2 separated by distance r, the potential energy equation becomes:

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right), \tag{1}$$

where G is the Newtonian gravitational constant, α is a dimensionless scaling factor corresponding to the strength of any deviation relative to Newtonian gravity, and λ is the characteristic length scale where deviations might occur.

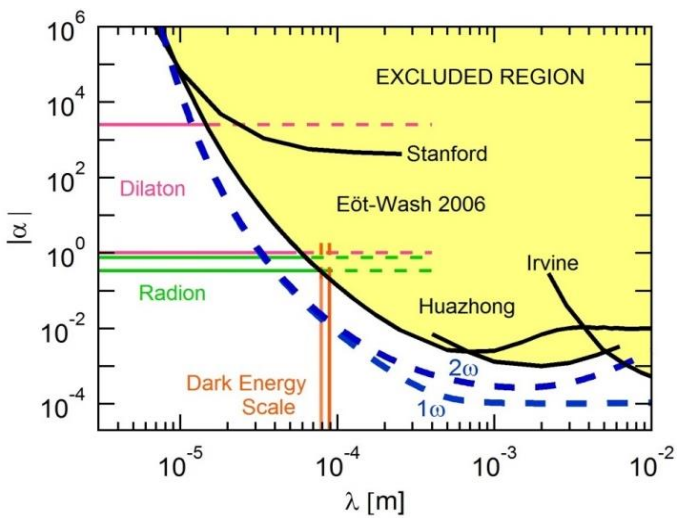

Figure 1: Current and predicted constraints on the ISL in the Yukawa potential parameter space.

Figure 1 shows current constraints on deviations from the ISL in α-λ Yukawa parameter space. The shaded corner has been excluded at a 95% confidence level by previous experiments[3,5,6,7]. The dashed line is the predicted sensitivity of the HSU experiment, which for some values of λ is an improvement from previous experiments by a factor of 50. Also shown are curves for various theoretical predictions that are described in Reference [4].

Previous experiments have tested and concluded, at the 95% confidence level, that α = 0[3,5,6,7] within experimental error bars, i.e., no deviation was found. This excluded portion of our parameter space is seen in Figure 1 as the shaded region, the unshaded region represents separations that have yet to the tested. Included in Figure 1 are some theoretical predictions of hypothetical particles that may mediate short-range forces and at what separations they are predicted to be found[4]. The dashed curves represent the range of α's and λ's that the HSU Gravity Lab will be

investigating. The HSU Gravity Lab experiment will be sensitive to smaller deviations at shorter separations. More information on the experiment's sensitivity will be discussed in Section 3.

3. Experimental Setup

The HSU Gravity Lab experiment is designed to test short-range gravitational interactions between the step-pendulum and attractor mass separated by 100 microns. At this separation, the experiment has intrinsic sensitivity to investigate gravitational strength effects with λ of 20 microns (in the same way that knowing the Earth's gravitational strength at the Moon, we can place upper limits on the size of any ISL deviation with λ of 1/5 of the Earth-Moon distance). An overview of the apparatus layout is shown in Figure 2. The HSU Gravity Lab uses a torsion pendulum to separate the experiment from the influence of terrestrial gravity. Our pendulum is suspended by a twenty-micron diameter tungsten fiber mounted on a magnetic damper to reduce any simple-pendulum oscillations caused by external disturbances. Given that the electromagnetic force is forty orders of magnitude stronger than gravity, the experimental setup must be shielded in a way to reduce electromagnetic systematic errors. An electrostatic shield made out of conducting BeCu foil (20μm in thickness) is stretched in a drumhead fashion on a machined support structure (this electrostatic shield not included in Figure 2). By placing the electrostatic shield between the pendulum and attractor mass, we can eliminate possible patch charge and Casmir effects. The apparatus is pumped down and maintained at 10^{-6} Torr to reduce viscous damping and thermal coupling to the pendulum. To thermally shield the entire experiment from the environment, the apparatus is housed in an enclosure made of insulating Styrofoam sheets.

A null experiment is achieved by noting, from Gauss' Law, that gravitational force for a test mass interacting with an infinite plane of matter does not depend on their separation distance. To exploit this fact, the experiment will use a parallel-plate configuration with a planar pendulum and a comparatively large attractor plate. The attractor-pendulum separation, *s*, is modulated by moving the attractor mass sinusoidally at angular drive frequency ω. The attractor modulation frequency is chosen to be different from the resonant frequency of the pendulum in order to ensure the measured angular twist will be due to the modulation of the attractor mass and not to resonant behavior of the pendulum. In Figure 1, the predicted sensitivity is seen as the dash curves 1ω and 2ω. As discussed in previous work[8,9], analyzing the twist oscillation amplitude at 1ω gives the most contaminated signal because the attractor plate is finite and causes residual Newtonian torque on the pendulum at that frequency. By analyzing the twist of the pendulum with Fourier techniques at increasing multiples of ω, we will reduce the amount of contamination from Newtonian torque and from environmental false effects.

Our "stepped pendulum" is composed of two materials of equal masses but with different density. Recently, we received our machined pendulum pieces: the Aluminum step and two Titanium blocks (shown in black on the right side of Figure 2). In the ideal case of an infinite attractor plate, the Newtonian torque on the pendulum does not vary with attractor position, while any short-range interaction would produce more torque on the closer high-density Titanium block when *s* is smaller than the range of the interaction. If the Titanium step were to fall toward the attractor mass more strongly, this would violate the WEP and introduce a variation in twist at 1ω or its harmonics.

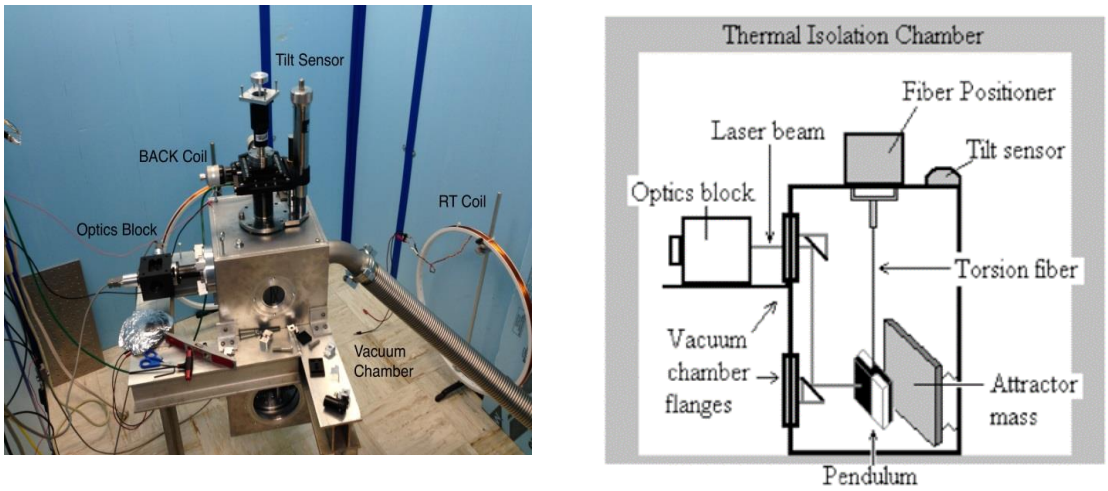

Figure 2: Left: Interior of Blue Box. Right: Experimental setup overview.

Figure 2: Left: inside of the thermal enclosure containing HSU Gravity lab's experimental apparatus.
Right: A schematic of the experimental setup: a laser enters the vacuum chamber through a window and is directed down to the pendulum via periscopic mirrors. The separation between the pendulum and the attractor mass varies sinusoidally at the angular frequency ω. The laser beam backtracks into the optics block where the twist of pendulum is measured by a position sensitive detector.

A null result would cause no observed twist of pendulum at the modulation frequency of the attractor mass or its harmonics. Therefore, any twist variations observed can be misconstrued as a deviation from the ISL and/or WEP. For this reason, additional tests will need to be conducted to determine the source of twist variations. To determine variations in the pendulum's twist, a gallium arsenide laser diode of wavelength 670 nm is driven at 500 Hz; this laser is reflected off the Aluminum mirror-finished side of the pendulum and back into the optics block that contains our precision autocollimator which allows us to analyze the output signal. In order to detect torque variations of gravitational strength, the optical system must be sensitive to torques on the order of 10^{-18} N·m which corresponds to a required angular resolution on the order of one nano-radian of uncertainty in a day's integration time. A position-sensitive detector (PSD) inside the optics block receives the returning beam then outputs four voltages proportional to the beam's location on the ten-millimeter-square PSD (OSI Optoelectronics model DL-10). Sum and difference voltages for each axis are amplified and sent to Stanford SR830 lock-in amplifiers before proceeding to the data acquisition board. The difference signal divided by the sum signal yields a measure of the angle in each of the two sensitive axes of the PSD; see previous work[8, 9] for a thorough overview of the HSU Gravity lab's optical system.

4. Discussion of Possible False Effects and Noise

The HSU Gravity lab has spent a great deal of effort in characterizing and/or suppressing sources of noise. Intrinsic instrumental noise from our optical system can limit the sensitivity to resolve angular twists of the pendulum. As discussed in Reference [9], we were able to lower the noise floor of our optical system to resolve one nano-radian of twist in one day of data integration time by implementing a polarizing beam splitter and a quarter-wave plate in the autocollimator system. In this paper we focus on other, environmental sources of error. Environmental false effects can overwhelm the signal, thus, it is of great importance to characterize any environmental source that would introduce an unwanted torque on the pendulum. Inside the apparatus, an electrostatic shield will be inserted between pendulum and attractor mass, all will also be gold coated to eliminate patch charges. To ensure temperature stability in the enclosure, the lab has installed a programmable thermoelectric temperature controller (TE technology, Model TC-24-25). In addition, four platinum resistance temperature detectors (PTRTD) will be placed to accurately assess temperature fluctuations inside the vacuum chamber, on the apparatus, and within the thermal enclosure.

4.1. Instrumental Noise

In addition to the discussion of instrumental noise sources in the lab's previous publications[8,9], we also have to consider the possible torque on the pendulum induce by the radiation pressure from the laser. However, the calculation below shows the torque applied due to radiation pressure would be very small. If the laser were to be off-center by 0.5 cm (a large overestimate), we see that the induced torque on the pendulum would be on the order of 10^{-19} N·m. This induced torque is a very small, and below our instrument's torque recording abilities (10^{-18} N·m).

To calculate the effect from the radiation pressure, we note that the PSD output is about 4.3V when we measure 50μW of optical power incident on the PSD. By taking this relationship and our analysis of the laser power noise, we can calculate the torque noise (τ_{rad}) associated with the laser pressure. If the pendulum is perfectly reflective, radiation pressure noise (P_{rad}) can be defined as two times the laser power noise (ρ_{laser}) divided by the area (A) multiplied by the speed of light (c). By manipulating the P_{rad} equation, we obtain an expression for the force noise (F_{rad}) to be used to subsequently determine the torque noise:

$$P_{\mathrm{rad}} = \frac{F_{\mathrm{rad}}}{A} = \frac{2\rho_{\mathrm{laser}}}{cA} \tag{2}$$

Laser power noise in the output of the PSD is $\approx 2.8 \times 10^{-4}\,\frac{\mathrm{V}}{\sqrt{\mathrm{Hz}}}$ at 500 mHz (1ω). Using the output voltage to Watts relationship above, $\rho_{\mathrm{laser}} \approx 4.60 \times 10^{-9}$ W. With the worst-case assumption that the laser hits the pendulum one laser beam diameter off center $|\vec{r}| = 0.5$ cm;

$$|\vec{\tau}_{\mathrm{rad}}| = |\vec{r} \times \vec{F}_{\mathrm{rad}}| = \frac{2\times\rho_{\mathrm{laser}}\times r}{c} \approx 1.53 \times 10^{-19}\,\frac{\mathrm{N\cdot m}}{\sqrt{\mathrm{Hz}}}. \tag{3}$$

To compute the angular noise ($\theta_{\mathrm{rad_{noise}}}$) due to radiation pressure fluctuations, we divide the torque noise by the torsional spring constant of the fiber, $\kappa = 6 \square 10^{-9}\,\frac{\mathrm{N\cdot m}}{\mathrm{rad}}$.

$$\theta_{\mathrm{rad_{noise}}} = \frac{\tau_{\mathrm{rad}}}{\kappa} = 2.55 \times 10^{-11}\,\frac{\mathrm{rad}}{\sqrt{\mathrm{Hz}}}. \tag{4}$$

The noise that would result from the laser hitting the pendulum off-centered is well below the electronic noise floor of our autocollimator ($3 \times 10^{-7}\,\frac{\mathrm{rad}}{\sqrt{\mathrm{Hz}}}$). The effect is further suppressed for two additional reasons; first, we have carefully lined up the laser to hit the center of the Aluminum step, at best we can ensure that $|\vec{r}| \ll 0.5$ cm. Secondly, our laser is chopped at a very different frequency (500 Hz) from the attractor mass modulation frequency (~500 mHz). Please see References [8,9] for additional discussion of instrumental noise in the optical system.

4.2. Magnetic False Effects

We carefully selected non-magnetic materials when designing our apparatus. However, there is a possibility that the materials selected contain magnetic impurities. As with all known sources of systemic error, we will exaggerate the sources of magnetic field and characterize its effect on the pendulum. More on the characterization of magnetic false effects will be discuss in Section 5.4.

4.3. Electric False Effects

Electric false effects can overwhelm our signal of gravitational strength at the desired distance. We will suppress electric false effects by gold-coating the pendulum, the electrostatic shield, and attractor mass to prevent the

coupling of patch charges to components of the apparatus. Also, the electrostatic shield will eliminate possible Casmir forces between the pendulum and attractor mass.

4.4. Temperature

Intentional thermal variations will be induced with the thermoelectric control unit to observe the response of the pendulum. The observed level of temperature fluctuations during normal operation multiplied by any observed feedthrough will yield a measure of any temperature-related false effect.

4.5. Apparatus Tilt

As the apparatus tilts, *s* will vary and any patch-charge interactions could preferentially apply a real or apparent torque to one side. Reduction of this false effect can possibly be attained by active feedback from the tilt sensor to the electrostatic control electrodes. The characterization of this feedthrough will be of paramount importance. The apparatus tilt will be purposefully exaggerated to measure the tilt-twist feed through.

5. Experimental Progress

Experimental progress has been made since the lab's most recent publication[9]. The PSD circuitry has been updated and installed. A method for working with the torsion fiber has been modified and refined. Temperature fluctuations within the thermal enclosure can cause systematic error in our data; therefore, a system of four PTRTDs will efficiently characterize these fluctuations to better understand its impact on our data. Lastly, two Helmholtz coils were constructed and installed in the enclosure to provide a varying magnetic field for magnetic systemic tests.

5.1. Torsion Fiber

Our pendulum is suspended from a thin torsion fiber. The torsion fiber must support the weight of the pendulum, as well as electrically connect the pendulum to the rest of the apparatus. The fiber is made from tungsten wire that is twenty microns in diameter, and the fiber is attached at both ends with copper screws. A major problem we encountered was in affixing the fiber to the screws, we cannot use adhesives because most adhesives are not conductors. The solution is to use a screw with a small hole through the center. The wire is passed through the hole and a section of the screw is crimped using a half-ton press. This crushes the copper with enough force so that the wire is secured and an electrical connection is made. Having very little mass, the fiber is thermally coupled to the walls of the vacuum chamber radiatively making it susceptible to temperature fluctuations of the vacuum chamber and of the isolation chamber. To attempt to reduce this error we are placing temperature sensors in and around the vacuum chamber and we are installing a copper shroud that will shield the fiber from infrared radiation coming off of the walls of the chamber.

5.2. Temperature Sensors

It is paramount to characterize the ambient environmental noise sources that may contaminate our data, especially their effect on the pendulum. We will characterize all known ambient environmental noises by exaggerating them (i.e. tilt, temperature, magnetic field) at the attractor modulation frequency and observing its effect on the pendulum.

Temperature fluctuations can cause expansion or contraction of the twenty-micron diameter torsion fiber. As we exaggerate temperature fluctuations within the thermal enclosure, we can then characterize the amount of temperature variation necessary to produce a measurable amount of twist on the pendulum. Four PTRTDs (Omega, Model 1PT100K2528) will be placed at various locations inside the thermal enclosure. Two sensors will be placed on the walls of enclosure while the other two sensors will be in and on the apparatus. This will effectively account for temperature changes while running the experiment. Circuitry was constructed to provide a one-milliamp constant current through the PTRTD to activate the sensing element. The circuit also scales (roughly $1\frac{V}{^{\circ}C}$) the output of the PTRTD giving us the ability to resolve milli-Kelvin temperature fluctuations at the attractor modulation frequency. This resolution will allows us to effectively determine impact on the data. The circuit will be printed on a circuit

board to have a compact package to place in the enclosure. Future plans for the temperature sensors are to have these sensors readings integrated with the existing data acquisition program to record temperature data alongside data collecting runs.

5.3. Systematic Magnetic Tests

Fluctuating magnetic field is another environmental noise source that can affect the twist of the pendulum. Using the exaggeration technique described in section 5.2, we can also characterize the effect of the ambient fluctuating magnetic field (Figure 4) on the pendulum. Two Helmholtz coils built out of copper wire and bicycle rims were constructed in-house to serve this purpose. After characterizing possible ambient magnetic noise, we can also determine the magnetic dipole moment of the pendulum using these coils. We send an oscillating voltage through a coil; as a result the Helmholtz coil produces an oscillating magnetic field at the location of the pendulum. By observing how big of an effect the fluctuating magnetic field has on the pendulum, we can then determine the magnetic dipole moment of our composition dipole pendulum and subsequently assess the level of magnetic systematic error. We applied a modulating external magnetic field at 10 mHz for the initial assessment of the Aluminum step's magnetic moment. A sample calculation of the magnetic moment of the Aluminum step is given in Section 5.3.1. previously we conducted a data run of the ambient magnetic field variations overnight. Figure 4 displays the Fourier spectrum of the ambient magnetic field at the site of the pendulum due to the Earth.

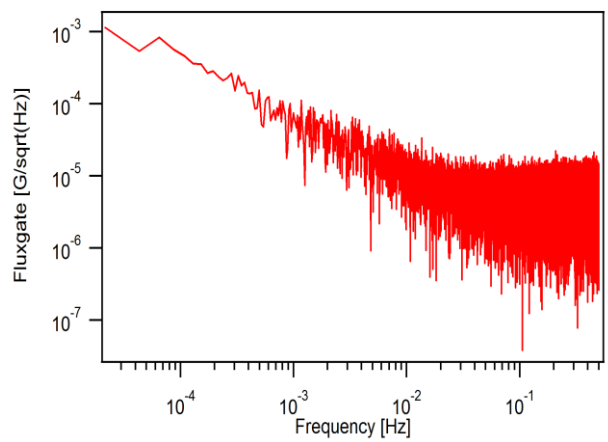

Figure 4: FFT of the Earth's magnetic field at the pendulum location.

Figure 4: Shows a Fourier spectrum of earth's magnetic field at the site of the pendulum. We can see the magnetic noise floor is $\sim 10^{-9}\,\frac{T}{\sqrt{Hz}}$ (B_{noise}).

5.3.1. Calculation of Magnetic Moment of Aluminum step.

The spring constant (κ) of our Tungsten fiber is approximately $6 \times 10^{-9}\,\frac{\mathrm{N{\cdot}m}}{\mathrm{rad}}$. The following numbers were obtained by running our data through our Python analysis program. The measured quantity θ_{amp} is the amplitude of the induce twist due to the applied oscillating external magnetic field of amplitude B_{amp}. By modulating the coil oriented in the North-South direction, we can measure the magnitude of the West-East component of the pendulum's magnetic dipole (m_{WE}) moment due to the fundamental relationship $\boldsymbol{\tau} = \mathbf{m}\square\mathbf{B}$. We found:

$$\theta_{\mathrm{amp}} = 1.4 \times 10^{-5}\mathrm{rad} \tag{4}$$

$$B_{\mathrm{amp}} = 9.8 \times 10^{-2}\mathrm{G} = 9.8 \times 10^{-6}\ \mathrm{T} \tag{5}$$

$$|m_{WE}| = \kappa \frac{\theta_{amp}}{B_{amp}} \approx 8.6 \times 10^{-9} \frac{N \cdot m}{T} \tag{6}$$

Thus, the West-East component of the magnetic moment ($|m_{WE}|$)is approximately $8.6 \times 10^{-9} \frac{N \cdot m}{T}$,we can calculate the North-South component of the magnetic moment ($|m_{NS}|$) of Aluminum step using the equations and magnetic amplitude from the right West-East coil, which gives

$$|m_{NS}| = \kappa \frac{\theta_{amp}}{B_{amp}} \approx 3.5 \times 10^{-9} \frac{N \cdot m}{T}. \tag{7}$$

Note that it is only horizontal components of the magnetic moment that can produce torque about the fiber axis. The total horizontal component of the magnetic moment $|\mathbf{m}_{total}|$ of the Aluminum step in units of $\frac{N \cdot m}{T}$ is:

$$|\mathbf{m}_{total}| = \sqrt{{M_{WE}}^2 + {M_{NS}}^2} \approx 9.3 \times 10^{-9} \frac{N \cdot m}{T}. \tag{8}$$

The calculated $|\mathbf{m}_{total}|$ is the magnitude of the magnetic moment of the Aluminum step alone, we expect this number to change slightly when we affix the Titanium blocks to the Aluminum step to complete our pendulum assembly. We can directly estimate the effect of this magnetic moment on the torque and angular noise of the pendulum:

$$\tau_{noise} = |\mathbf{m}_{total}||B_{noise}| = 9.3 \times 10^{-18} \frac{N \cdot m}{\sqrt{Hz}} \tag{9}$$

$$\theta_{mag_{noise}} = \frac{\tau_{noise}}{\kappa} = 1.55 \times 10^{-9} \frac{rad}{\sqrt{Hz}} \tag{10}$$

The magnetic noise ($\theta_{mag_{noise}}$) is also below the noise floor of autocollimator; therefore, an induced twist in the pendulum from ambient modulating external magnetic fields will not have an influence on our data collection for this pendulum.

6. Conclusions

We are developing an experiment at Humboldt State University that will probe the gravitational ISL and the WEP at untested distance scales. The novel parallel-plate torsion pendulum design will be the most sensitive apparatus to date for measuring gravitational effects at 20 micron separations. With the apparatus completion in sight, the lab is expecting to start taking data soon. We have begun to address an important goal of understanding and characterizing environmental noise contamination in our data.

7. Acknowledgements

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