Proceedings of The National Conference
On Undergraduate Research (NCUR) 2014
University of Kentucky, Lexington, KY
April 3-5, 2014

Interpreting Audible Intervals into Visual Learning Tools

Caitlin Cowen
Music - General Studies
Nicholls State University
Thibodaux, Louisiana 70301 USA

Faculty Advisors: Dr. Carol Britt, Mrs. Trisha Rabalais, Dr. Rodney Soigner

Abstract

Sounds are intertwined in a mathematical network, and this project attempts to visualize the exponential factors used in determining musical frequencies. Considering that consecutive sounds are related mathematically, particularly musical sounds constructed on multiplicative ratios, could the visualization of sounds represent their mathematical relation? Of course representing sounds in a visual format is not a novel concept as there are many forms of notation, particularly for musical sounds. However if traditional musical notation is filled with visual discrepancies, is it an accurate visual representation? This project questions the visual accuracy of popular musical notation. Furthermore an alternative notation was created and compared to traditional music notation. In a small study, the alternative notation has proven to be more accessible to non-musicians while remaining accessible to musicians. While this alternative is not meant to replace traditional notation, as visualizing rhythm has not been attempted, it could be used as an interactive tool for those without musical knowledge. Also, the basics of the alternative notation are outlined.

Keywords: Music Cognition, Visual Communication, Ear Training

1. A Brief Look at the Origin of Musical Tones and its Notation

Some 2,500 years ago, scholar Pythagoras passed by the blacksmith's shop; the sounds produced by swinging hammers in the shop inspired Pythagoras to investigate their relations. He concluded hammer weights related by simple integer ratios produced harmonious sounds.[1] If the hammer was twice as big, it would produce a sound an octave higher, and if the hammer was three times bigger, it would produce a sound an octave and a fifth higher.[2] Intervals can be related to simple ratios, multiplied upon the original frequency to produce new tones. Each interval has a unique harmonic relationship. The most basic ratio is to double or halve the tone. This raises or lowers the note by an octave. This ratio is found in virtually all cultures, and can even be recognized by some animals.[3] Though these notes are not the same frequency, they sound alike and the same letter identifies them. In standard notation, the note "C" an octave above is a "C' ". The note "A" an octave below is an "A' ". To speed the learning and understanding of intervals, they are not thought of as factors of multiplication. Intervals are understood additively. An octave plus an octave equals two octaves. In other words 1+1=2. However to sound the frequency two octaves above the starting note, the original frequency is multiplied by 2 twice. If the original frequency is 1, the frequency an octave above it would be 2. Two octaves above it would be 4. In other words 1x2x2=4.

The next basic ratio, multiplying or dividing by three, raises or lowers the tone by an octave and a perfect fifth. Continuing to raise notes by perfect fifths ultimately led to the musical notes known today.[4] Figure 1 graphs note 1 multiplied by three twice, creating *so* then *re*. The first column displays successive octaves additively. Each additional octave is added by an integer of 1. The second column represents reality, the octave's frequencies; starting from 1, each successive note is multiplied by 2. The column to the right multiplies the previous column's frequency by 3. Note that within each column, dividing or multiplying by two changes the note by an octave. The diagram only illustrates *do* x 3 = *so*, *so* x 3 = *re*, but theoretically the graph is endless. Because *do'* is only approximately the same through consecutive simple integer ratios, instruments today are not tuned exactly to these simple ratios. Instead equal-tempered tuning is used today. Notably before equal-tempered tuning was officially adapted, instrumentalists created systems quite like equal-tempered tuning.[5] Johann Sebastian Bach wrote *The Well Tempered Clavier* in 1722 specifically to call attention to equal tempered tuning.[6] By composing 24 pieces, one in every major and minor key, the compositions could not be played together without equal-tempered tuning.

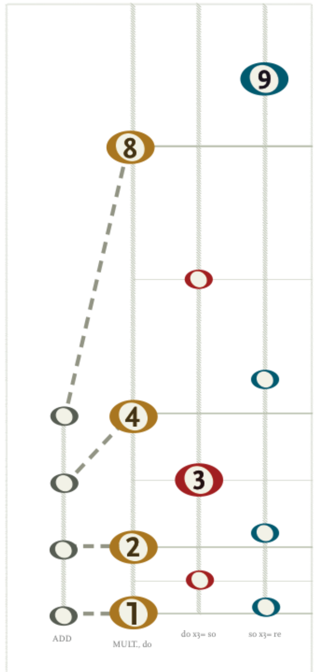


Figure 1: *Do* times three equals *so*, *so* times three equals *re*

2. Visual Rules in Comparison to Music Notation

Music notations are data graphics by comparison to the following definition:

> "Data graphics visually display measured quantities by means of the combined use of points, lines, a coordinate system, numbers, symbols, words, shading, and color."[7]

Sounds, particularly musical, are measured quantities that in traditional Western music notation are represented with symbols on lines roughly representing a graph or coordinate system. This notation represents two dimensions: pitch and time.[8] Thus according to definition, music notation may be considered a data graphic. The y-axis records pitch height, which can be modified by additional symbols. The y-axis consists of any number of five-lined groups, called staffs. Two lines create three possible resting places for symbols on three pitches. There are two lines, but three spaces; visually 1+1=3.[9] A five-lined staff creates eleven possible resting places for symbols on eleven pitches, and can be temporarily extended with ledger lines. The x-axis is the marker of time, whose scale is controlled by symbols. The x-axis serves as a reminder of the passing of time, but does not constrain notes to equal scaling in time markers called measures.

If music notation can be considered a data graphic, then it can be compared to the following principles for data graphics, numbered for reference.

> "Excellence in statistical graphics consists of complex ideas communicated with clarity, precision, and efficiency. Graphical displays should: [1] show the data [2] induce the viewer to think about the substance rather than about methodology, graphic design, the technology of graphic production, or something else [3] avoid distorting what the data have to say [4] present many numbers in a small space [5] make large data sets coherent [6] encourage the eye to compare different pieces of data [7] reveal the data at several levels

of detail, from a broad overview to the fine structure [8] serve a reasonably clear purpose: description, exploration, tabulation, or decoration [9] be closely integrated with the statistical and verbal descriptions of a data set. Graphics reveal data." [10]

Principles 1,2, 4, 6, 8, and 9 are exceptionally employed by traditional Western music notation. Perhaps the notation's greatest strength follows principle 6's instructions. The eye can easily compare the pieces of data representing pitch height, which as a group of vertical changes represent the contour of a song, Familiar melodies can be easily identified by similar contours, 'the up and down pattern of pitch changes'.[11] The ease of audibly recognizing similar contours matches the simplicity of its visual representation in traditional Western music notation. Contour is so easily grasped by listeners that it can be given another dimension in word painting. Predictable examples of word painting are words ascending and descending associated with musical scales played up and down.[12] A visual parallel to word painting is evident in Figure 2. The rising notes represent the action of a rising soul into heaven, directly comparable to the pictured ascending notes into the clouds. The link between physical space and musical pitch is undeniable in traditional Western music notation, and an asset to the system as a data graphic.

Figure 2: Johann Jakob Froberger, Suite XII in C major, *Lamento sopra la dolorosa perdita della Real Msta di Ferdinando IV, Re dé Romani*[13]

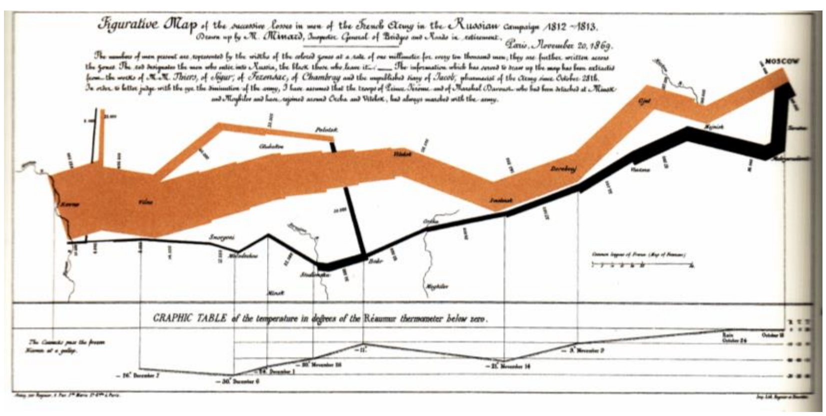


Figure 3: English translation of item 28 from *Tableaux Graphiques et Cartes Figuratives de M. Minard* by Charles Joseph Minard [14]

However traditional Western music notation may not fulfill principle 3 that in turn leads principles 5 and 7 astray. In comparison, Figure 3 is one of the greatest statistical graphics in history. It is a visual record of the French army's invasion into Russia, recording the army's population and path. Here every 10,000 men is represented with a millimeter of space on the colored path. It is an exceptional example for principle 3; the space equally represents a quantity of data, 10,000 men. If Figure 3's scale changed on Wednesdays to 5,000 men per millimeter of space, an accurate interpreter must have previous knowledge of the scale change and a list of Wednesday's dates for this year. The average reader may not be capable of accessing the information of this hypothetical, skewed graph. This is similar to traditional Western music notation, in which each ledger line represents one step of the major scale. The space equally represents a quantity of data, a step in the major scale. However that is the misrepresentation in question as the steps of the Western major scale are not equal. In fact unequal step scales, like the major scale, have a processing advantage as compared to equal-step scales.[15] The Western major, or Ionian, scale's pattern consists of whole (W) and half (H) steps; its pattern is W-W-H-W-W-W-H.[16] A half step is naturally half of a whole step. Therefore most ledger lines represent whole steps while select few represent half. Traditional Western music notation is comparable to the hypothetical description of Figure 3 when its scale changed from 10,000 men to 5,000 on Wednesdays. In the hypothetical example and in traditional Western music notation, an accurate interpreter has previous knowledge of the scale change and a directory of where the changes occur. The average reader may not access the information of the hypothetical, skewed graph or traditional Western music notation.

If indeed principle 3 is violated in traditional Western music notation, principles 5 and 7 are affected. Principle 7 is partially followed. From a broad overview similar contours can be associated, but the fine structure, the differences of pitches by a semitone, may be skewed. Principle 5 is also partially followed; large data sets are logically connected, but sometimes disconnected though only by a semitone.

For example, the same vertical distance can be visually represented for half and whole steps. In musical terms, *Mi* to *fa* (E-F) is a half step; *ti* to *do* (B-C) is a half step. Everything else (*do* to *re* [C-D], *re* to *mi* [D-E], *fa* to *so* [F-G], *so* to *la* [G-A], *la* to *ti* [A-B]) is a whole step. See Figure 4. Without previous musical training, a reader could not determine which intervals are half steps; the data graphic alone does not visually indicate these variations. For instance different fourths and thirds can be seen as the same, but heard differently. In Figure 4 an augmented fourth, also known as the Devil's interval or tritone, is certainly not the same as perfect fourth, though at times they are visually represented with the same space. Also in Figure 4 a minor third and major third are not the same though they appear to be.

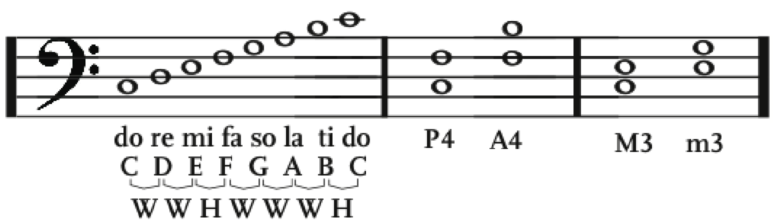


Figure 4: Examples of data misrepresentation

Musicians have allowed this system to work by memorizing the Ionian, major scale while recognizing accidentals to be the irregularities. With the use of key signatures these scales can be transposed without a visible change; every Ionian, major scale with a key signature lacks accidentals. Figure 5 depicts a major scale in different key signatures. Put your hand over the key signatures. Notice how the sequence of circles does not appear to change by use of the key signature.

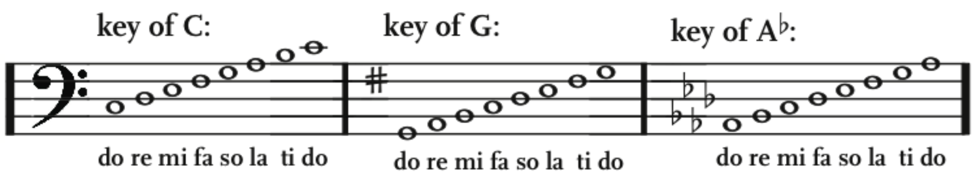


Figure 5: Three major scales in different keys

Discrepancies in another music notation, numbered notion, stem from the same assumption- each step of the scale can be recognized equally. Though the scale is a series of alternating interval steps, there is a problem distinguishing these slight nuances. For instance in numbered music notation employed by the Chinese, each of the different steps of the scale is equally recognized by whole number values.[17] In this notation with each successive note of the scale, the integer increases by 1. However mathematically if a whole step is equal to 1, a half step should equal 0.5, half of a whole step. To relate numbered notation to solfege syllables, if the initial note is *do* is equal to 1, *re* is 2, *mi* is 3, *fa* is 4, *so* is 5, *la* is 6, and *ti* is 7.[18] This would bring *do*' to 8. However if half steps were accounted for, *do* would equal to 1, *re* is 2, *mi* is 3, *fa* is 3.5, *so* is 4.5, *la* is 5.5, and *ti* is 6.5. This would bring *do*' to 7. In another variation *do* could begin on 0; then *re* is 1, *mi* is 2, *fa* is 2.5, *so* is 3.5, *la* is 4.5, and *ti* is 5.5. This would bring *do*' to 6, and since there are six whole steps in an octave, the set of numbers truly add up. Current numbered notation and traditional Western notation do not add up; thus music math does not work like basic math. A perfect fourth plus a perfect fourth is a minor seventh. A major third and minor third add to a perfect fifth.

3. Study: Identical Data in Two Data Graphics, Traditional Western Music Notation versus Circle Music Notation

If traditional Western music notation is a data graphic, choosing certain sound frequencies to be heard in a specific timeframe, then similar data can be visually linked by readers. This could be tested in a multiple-choice format. If the same musical data were presented to readers in two different data graphics, would either data graphic's information prove to be more accessible? Does a reader's musical experience affect their success in interpreting either data graphic?

3.1 Method:

Participants ranked their musical experience and were presented with eight questions, preceded by instructions and two example questions. The eight questions are more accurately described as four questions presented in two different formats. Questions 1-4 asked readers to match data presented as traditional Western music notation (TN), a format more familiar to those with musical experience. Questions 5-8 paralleled the musical data presented in questions 1-4 in circle music notation (CN), a format new to all participants. The study does not question the rate of accuracy for TN, but specifically pulled data to purposely compare questionable data graphics. There were 83 participants collected from 7/30/14-8/13/14. Four surveys were thrown out because they did not rank their musical experience.

The questions were in multiple-choice format. All questions pictured two simultaneous notes, except question 4 (and therefore 8), having three notes. In TN note stems, time signatures, and vertical bar lines were eliminated to avoid confusion. Participants were asked to determine the difference between the notes, aka the interval (major second, octave, etc.) to verify the answer, as one of the multiple choices represented the same interval and was the correct answer. It is like the picture said 4 and 8. A correct, matching answer would be 10 and 14, but a wrong choice is 10 and 13. See Figure 6 for a sample question; notably parallel data is represented in TN and CN. The figure pictures separate questions, 2 and 6, together; participants did not view questions 2 and 6 together.

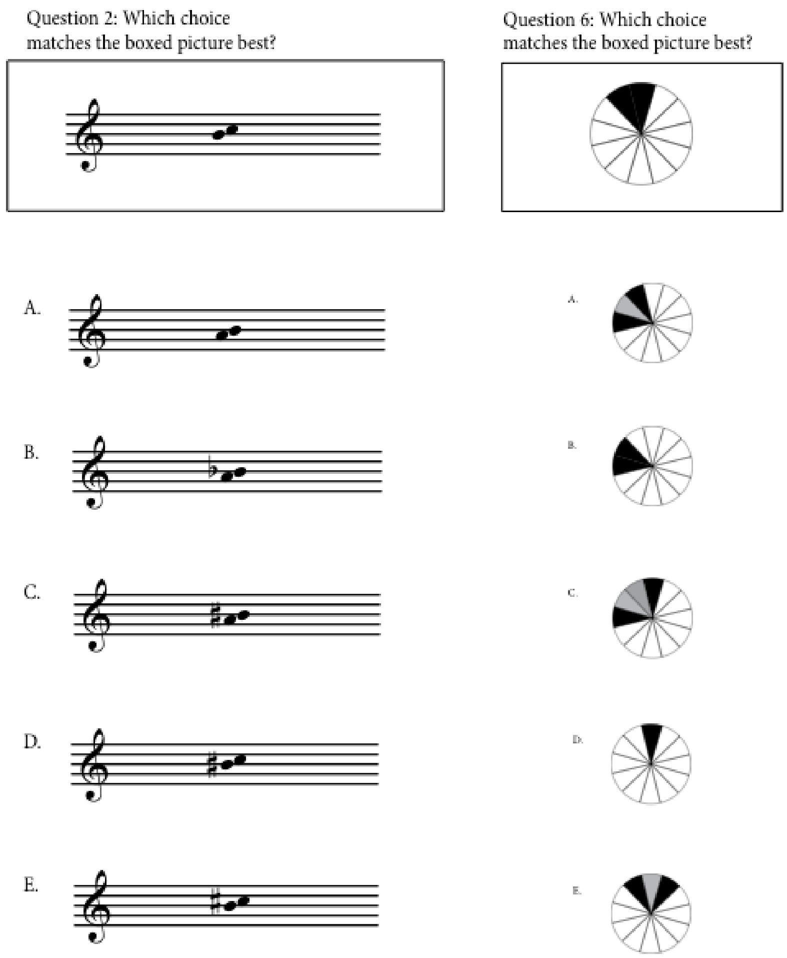


Figure 6: Sample questions 2 and 6; the correct answer is B.

3.2 Data Analysis:

The results of the present study are summarized in Figure 7, which indicates that for all but the most highly trained musicians, interval identification is superior for CN than TN. The data was examined for outliers and none were found. The results for all valid cases were analyzed using Graphpad Prism for Windows (Version 5.01). A 3 x (2 x s) mixed factorial ANOVA (Level of Education x Notation Type) revealed a significant Interaction between the variables [F (2,76) = 40.85, $p < 0.01$]. Post Hoc analysis using Tukey's LSD test determined that CN was superior to TN for novice and moderate levels of musical education, but not for experts. No other significant results were obtained. Interaction accounts for 5.13% of the total variance. F = 40.85. DFn=2. DFd=76. The P value is <0.0001. If there is no interaction overall, there is a less than 0.01% chance of randomly observing interaction in an experiment of this size. The interaction is considered extremely significant. Since the interaction is statistically significant, the P values that follow for the row and column effects are difficult to interpret.

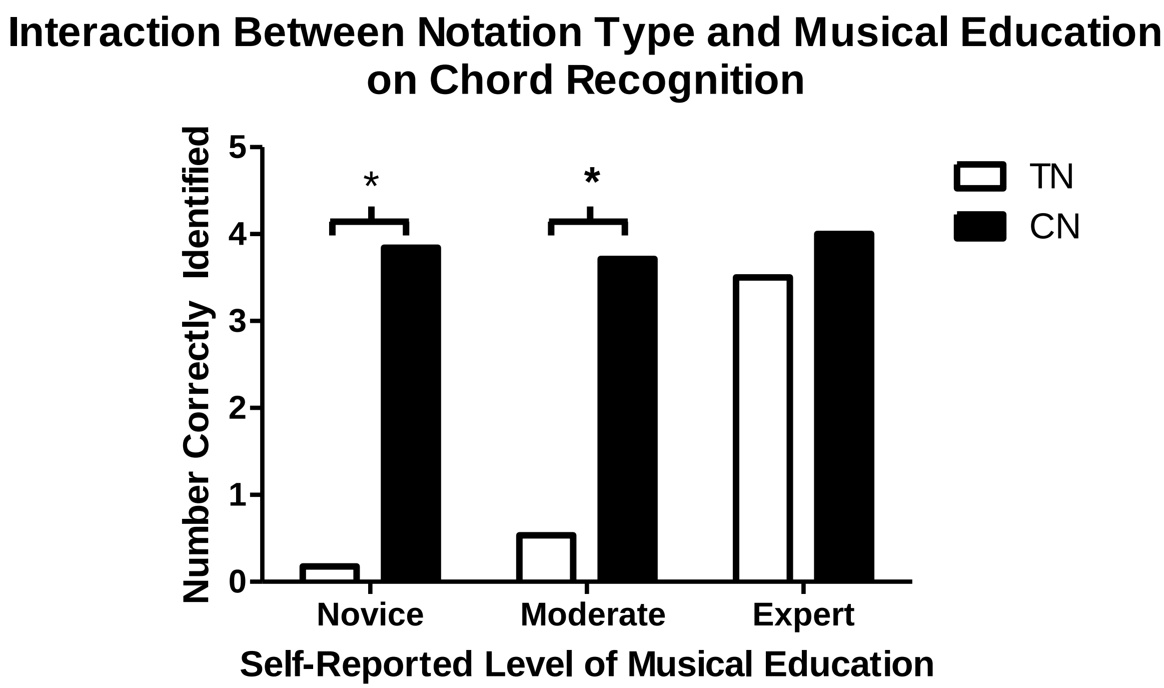


Figure 7: Results of present study

3.3 Observation And Discussion:

Many people commented upon the easy nature of the second section (CN). Participant 51 said he had no idea what he was doing for TN and CN was much easier. Participant 55 said the circles were architectural, even fun, and 'could do this all day'. Participant 63 wrote none for questions 1 and 3, and close for 4; these were all TN. Questions 5-8 (CN) were all answered and correct. Participant 83 spent twice as long on TN as opposed to CN; she even went back to TN after completing CN. She missed all TN and correctly answered all CN. Participant 3 was a musician who could have calculated the answers for TN, but grew tired of the process; she finished two of the four TN correctly while she guessed incorrectly on the other two questions.

Participant 65 was the youngest participant at age 9. She took piano lessons and was familiar with reading music. She said her logic in testing for TN was 'If the boxed picture doesn't have a sharp or flat, the answer shouldn't have a sharp or flat.' This simple logic could have governed many participants' decisions as many answers paralleled the child's answers. Like the girl, 59% of participants answered A for question 1; 69% answered A for question 2; 57% answered B for question 3. When the youngster was presented with CN, she assigned a colored fruity flavor to each gray shade of a delicious pie. She correctly identified 2 of the 4 questions for CN; notably these questions were smaller intervals. With larger intervals she choose similar data by the direction the interval faced. Participants were 9% more likely to miss the larger intervals in CN, questions 7 and 8, as opposed to smaller intervals, questions 5 and 6. Other design elements, most obviously color, should be added to assist identifying larger intervals.

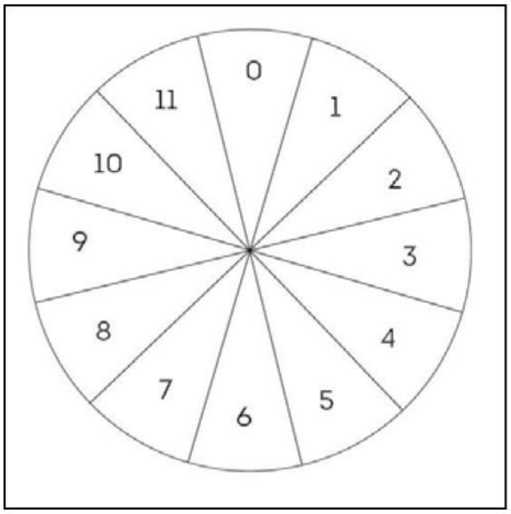
Participant 5, a pianist, could not correctly answer any TN questions until he pulled up a picture of a keyboard on his phone. Then he got questions 2 and 3 right. Though he could identify note names from TN, he could not determine the intervals without the additional visual of the keyboard and said he became less distracted with the additional visual. The visual did not include note names. He counted the distance by white and black keys. In the opposing CN section, he said he did not have to think about answering CN; it was basic. A few instrumentalists ranked as moderate musicians identified note names. However identifying notes is not enough to calculate the intervals between them. While this calculation is not necessary to play an instrument, the ability to recognize intervals is critical to relative pitch perception, allowing melodies to be recognized in different keys.[19] If most people have relative pitch perception[20], could a data graphic reinforce this ability?

Contrary to the above participants, people in the elite musician group can answer TN correctly, even easily. The elite musicians are hard to find and are easily the smallest group in the study. These participants can decipher specific technical information from TN. Participant 1 flew through both TN and CN with a perfect score. Participant 4 was an elite musician who flew through the test, but missed question 4 because she overlooked a sharp symbol in the key signature. Perhaps the key signature can be overlooked because it is temporarily stored knowledge for instrumentalists. When a key signature is presented, the instrumentalist makes a mental note to extend the modification to selected notes. Therefore visually the note alone is not enough; it is misrepresented though only by a semitone. If indeed the note is misrepresented on the y-axis (recording pitch height) through the use of key signatures and furthermore other symbols such as sharps and flats, then maybe instrumentalists typically read through misrepresentations. If it is common practice for music readers to internally modify visual data, could this be the reason for participant 4's mistake? Can musicians read through mistakes easily? Musicians are known to read though misprints; this was studied by Slobada who was inspired by Goldovsky.[21] Competent pianists were presented with lesser known classical pieces filled with misprints. The misprints intentionally displaced the note by a semitone so that it was harmonically out of character. Pianists read over misprints (correcting them), and even more so on the second performance. The experiment suggests that musical knowledge is entangled with musical reading.[22] How useful is TN as a data graphic if misprints are read over by musical knowledge? Do musicians rely on more on TN or their own minds? If TN was equally represented would musicians rely on what they see or what they know? These questions are far from answered; this study may merely open doors for further questions. Though reading music is a necessity to join a musical world, teachers, educationalists, and psychologists have paid little attention to music reading.[23] Can this study call attention to music reading and furthermore open a musical world to a wider range of people?

4. Basics of Circle Notation, patent pending

Sequential audible information needs to be viewed and interpreted visually to further the human understanding of its relationships. If the visual notation of audible sequences is misleading, perhaps it undermines learning, such as ear training and accurate pitch placement. Simply stated, current musical notation may not present notes, representing frequencies, in a way that accurately illustrates the mathematical relationships between them. This is because popular musical notation is an uneven graph of one-dimensional lines. Horizontal lines create vertically incremented spaces that mostly represent whole steps, while select few signify half steps. Therefore same sounding intervals are represented with different vertical distances, making transposition and inversions unreasonable. Also symbols (ex: sharps and flats) designating differences in pitch reject their vertical location.

Thus the following invention was created to attempt to surpass existing systems. It is a circular visual system to display or represent sequential audible information through mathematical relationships, represented in a continuous overlapping circle, adapted to represent a sequence of frequencies in one octave to each 360-degree rotation. This new system differs because it is a two dimensional shape in link with a sequence, allowing a new visual relationship to exist between humans and audible information. Because it is a two dimensional shape, created from mathematical relationships, the system consistently represents intervals in the same, predetermined relationship. These relationships are easily paralleled in visual rotation or reflection, allowing for easy transposition and inversion, or even both at once. Figure 8 depicts a simple diagram for the system. Further reading is more detailed.



0	C
1	C#
2	D
3	D#
4	E
5	F
6	F#
7	G
8	G#
9	A
10	A#
11	B
12	C’

Figure 8: Simple diagram for circle notation, key of C, labeled with sharps

4.1 Steps:

[1] Divide a circle equally like pizza slices into the number of variables in one steadily increasing or decreasing sequence; the sequence ends when the starting variable has come to a mathematical equivalent. If adapted to the Western musical scale, divide into twelve slices. The sequence is the number of frequencies recognized in one octave; when a full rotation is completed, the starting frequency has doubled or halved, its mathematical equivalent. This sequence, whose frequencies steadily change, is C, C#, D, D#, E, F, F#, G, G#, A, A#, B. They are also enharmonically named C, Db, D, Eb, E, F, Gb, G, Ab, A, Bb, B, among other spellings.

[2] Assign a numeric variable to each slice in a clockwise direction starting with zero. Continue numbering in a clockwise direction though pieces will have multiple numeric variables. Also continue from zero in a counterclockwise direction allowing negative integers. This is mainly for background identification, so they will not be visible in the final product.

[3] Assign a name to each variable. If adapted to the Western musical scale, 0=C, 1=C#, 2=D, 3=D#, 4=E, 5=F, 6=F#, 7=G, 8=G#, 9=A, 10=A#, 11=B. Continuing the sequence onto every numeric variable, 12 is the C above the first C, as -12 is the C below the first C. 13 is the C# above the first C#, as -11 is the C# below the first C#. Theoretically this process is infinite. Every slice represents the same name in the sequence many times by continually overlapping variable names.

[4] Assign a visual and/or audio output to each variable upon interaction with the slice. The shape may be modified to support user interface. If adapting to the Western musical scale, according to the assigned variable names, the variables could approximately correlate to the well-tempered scale. Thus the variable names A could approximately equal 21=A=880 Hz, 9=A=440 Hz, -3=A=220 Hz. Simple integer ratios could be calculated. Existing equal-tempered scales, among other scales, may be used. The process of assigning audio output is endless as is the assignment of numeric variables defined in step (2) and variable names defined in step (3). Visual output can be in color changes. A pattern apt to the goal at hand may be assigned to receive special designation. If adapted to the Western musical scale, perhaps the slices of the major scale are green in compliance to many songs. Upon interaction the slice can be assigned another color. If adapted to the Western musical major scale, while the designated major scale pattern is green, the active slice/s is/are yellow.

[5] Upon interaction with individual slices or input into the system, visual and/or audio output is executed.

The initial step [1] divides the shape into the number of variable pieces needed. The next step [2] links numeric variables to the slices for background identification and reference. Step [3] assigns a variable name to each numeric variable for public identification and reference. Step [4] assigns a visual and/or audio output upon interaction or input into the system. This is the display or representation of audible information allowing a new visual relationship to exist between humans and audible information.

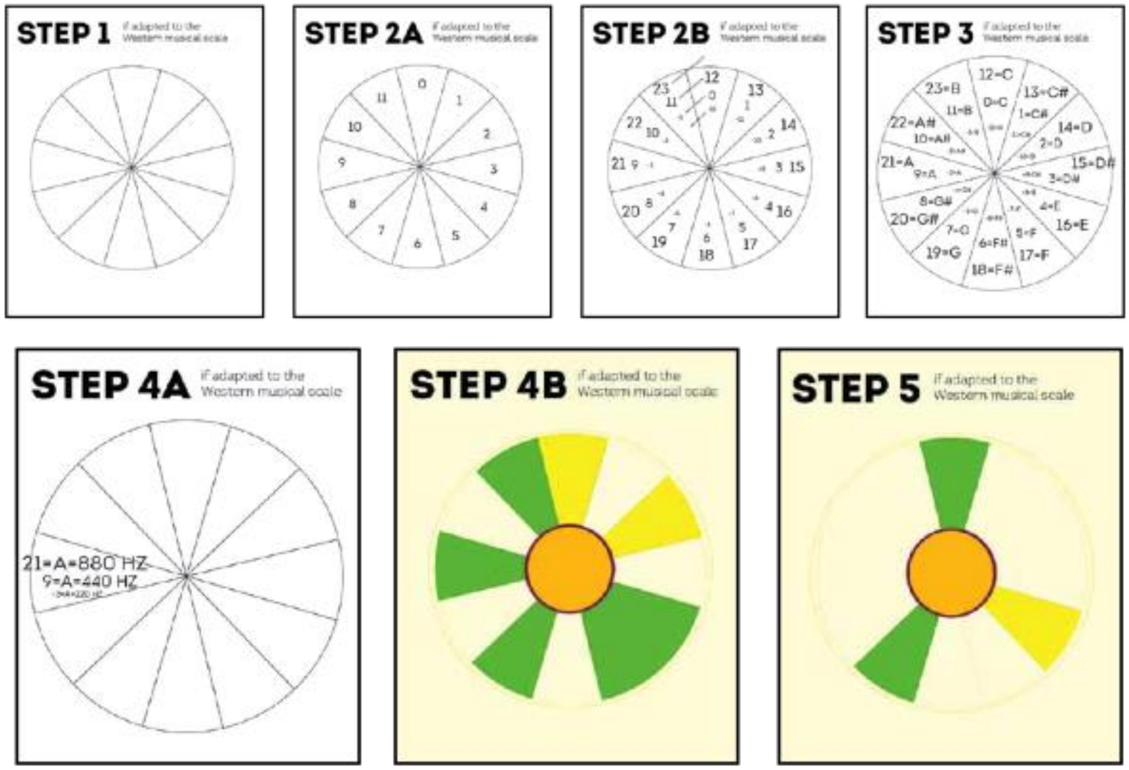


Figure 9: Steps 1-5

4.2 Furthermore:

By following the above steps, a visual information system is created for steadily incrementing sequences. It links given variables to seen and/or heard output. Interactions can be through click or touch; input can also be generated without interaction. It can be physically created in an independent device. It can also be created in mediums like but not limited to a computer or tablet.

It is necessary to have a space that can be divided into a sequence of variables. The invention can take any form that provides a two dimensional space. The numeric value assigned to the divided spaces is absolutely up to the creator's discretion as they are mainly for background identification. For example if the creator does not want to start at zero or name in increments of one, they may do otherwise. Repetition of numeric variables in one space varies. Variables may have two names that can be interchanged or enharmonically named. It is optional to assign a visual or audio output to each division. It is optional to scale each division to its variable's equivalent, changing the space's dimensions. Adding outputs that would impact other senses, such as touch, could expand the invention. If the creator is satisfied with the system before reaching the last step, they may stop. If the creator wishes to skip any step, they may. The steps also do not have to be executed in order. The initial shape chosen is not limited to a circle. It can be any two or three-dimensional shape; this includes but is not limited to squares, triangles, discs, cubes or cones. It is preferable to equally divide the space, but select variables may receive larger or smaller divisions. The space does not have to be divided like pizza slices. The variables may rotate within the shape during use. Furthermore the entire shape may rotate. The given example of the Western musical scale is only one adaptation of the system.

Other scales with more or less notes in one octave, or other mathematical equivalent, can fit the same system. Also two, or any amount, of octaves may be assigned to each 360-degree rotation. Any interval may be assigned to each 360-degree rotation. Furthermore the system may be adapted to uses outside the musical world.

The length of audio/visual input may be experienced in real time or physically notated on the shape. This means it may be notated with another visual output- a symbol or in a predetermined shape change. For instance the longer the visual/audio output, the longer the piece may extend from the center. As time passes the shape may retract back into the center. When the audio/visual output has finished, the movement has stopped.

For large audio tracks involving many instruments, multiple circles may be assigned for different instruments. A single instrument may be separated into two circles. For instance in a piano piece, two circles may represent the two different hands of the pianist. Also for large audio tracks involving many instruments, different instruments can receive different visual outputs, colors or symbols. Multiple circles may be layered atop each other.

The invention may be applied in these ways. Knowledge of the sequence of variables is obtained by observing the changes caused by input into the system. If adapted to a musical scale, input into the system refers to playing a saved audible sequence or song. The track or song would link visual and/or audio output to the system's interface. The system may record interactions with the user. If adapted to a musical scale, this would serve as a means of composition. The system poses questions requiring interaction with the user. In this training they learn the correct answer through classical conditioning. This may be referred to as cognitive game training. By means of these and/or other questions or games, the system can also test and provide an assessment of a user's knowledge of the sequence of variables. The system may provide visual and/or audio output by 'listening'. If adapted to a musical scale, the system would parallel generated sounds, perhaps by a piano or other instrument, into the system's audio/visual output. Thus the audible information would be composed on another instrument and generate the same output as a saved song. This process may be combined with other uses of the invention. Furthermore, as many as all or as few as one of the above processes may be combined for the invention's use.

5. Acknowledgements

The author would like to express her appreciation to the Nicholls State University's Honors Program. She would also like to thank faculty research advisors Dr. Carol Britt and Mrs. Trisha Rabalais for their guidance. Special thanks to Dr. Rodney Soigner for his help in the statistical analysis.

6. References

1. Johnston, Ian D. *Measured Tones: The Interplay of Physics and Music*. Boca Raton: CRC Press, 2009, 5.
2. Ashton, Anthony. *Harmonograph: A Visual Guide to the Mathematics of Music*. New York: Walker, 2003, 4.
3. Levitin, D.J. & Tirovolas, A.K. (2009). Music cognition and perception. In Sage Encyclopedia of Perception. (pp. 599-606). Thousand Oaks: Sage Publications, 601.
4. Fauvel, John, Raymond Flood, and Robin Wilson J. *Music and Mathematics: From Pythagoras to Fractals*. Oxford: Oxford University Press, 2006, 20.
5. Ibid. 1, 74.
6. Ibid.
7. Tufte, Edward R. *The Visual Display of Quantitative Information*. Cheshire, Conn. (Box 430, Cheshire 06410): Graphics Press, 1983, 9.
8. Ibid., 4, 96.
9. Albers, Josef. *Search versus Research*. Sl.: Sn., 1969, 17.
10. Ibid., 7, 13.
11. Palmer, C. & Junger, M.K. (2003). Music cognition. In L. Nadel (Ed.), Encyclopedia of Cognitive Science, Vol. 3. (pp. 155-158). London: Nature Publishing Group, 155.
12. Taruskin, Richard. *The Oxford History of Western Music*. New York: Oxford University Press, 2010, 236.
13. Ibid. 4, 92.
14. Tufte, Edward R. *Beautiful Evidence*. Cheshire, CT: Graphics Press, 2006, 122.
15. Trehub, Susan E. et al., (1999). Journal of Experimental Psychology: Human Perception and Performance Vol. 25, No. 4. (pp. 965-975). American Psychological Association, 969.
16. Brofsky, Howard, and Jeanne Bamberger Shapiro. *The Art of Listening; Developing Musical Perception*. New York: Harper & Row, 1969, 143.
17. Alan R. Thrasher, et al. "China." Grove Music Online. Oxford Music Online. Oxford University Press. Web. 27 Apr. 2014. <http://ezproxy.nicholls.edu:2450/subscriber/article/grove/music/43141pg2>.
18. Ibid.
19. Ibid. 3, 604.
20. Ibid. 11.

21. Sloboda, John A. *Exploring the Musical Mind: Cognition, Emotion, Ability, Function*. Oxford: Oxford University Press, 2005, 57.
22. Ibid. 21, 58.
23. Ibid. 21, 3.