Optimal allocation of employee resources to meet staffing demands

Eli C. Towle and Kane P. Mach
Department of Mathematical Sciences
University of Wisconsin - Stevens Point
2100 Main Street
Stevens Point, WI 54481

Faculty Advisor: Dr. Andrew J. Felt

Abstract

Throughout each summer, the University of Wisconsin - Stevens Point plays host to a variety of conferences. Each week, staffing needs for ten residence halls can change dramatically, depending on the duration and scope of each conference. Motivated by the inefficiency and marginal inaccuracy of creating schedules by hand, the summer conference coordinator inquired if a mathematical model could be built to automate and optimize the staffing process. As a result, a mixed-integer linear program was produced using AMPL (A Mathematical Programming Language). It is capable of better utilizing the staffing resources available to the university by accurately meeting all staffing requirements stipulated by the summer conference coordinator. These include daily and weekly hour restrictions, honoring approved individual requests for time off, and limiting the number of shifts for each worker. The most important parameter is ensuring every hall is properly staffed at each hourly interval. The solution was a great improvement to previous schedules generated by hand, as evidenced by the objective function of the program. Overall, the mathematical model met all requirements while successfully minimizing the number of scheduled secondary employees, reducing the total number of double shifts, and distributing hours evenly. This model can be applied to other institutions or businesses where specific amounts of workers are required at different time intervals throughout the day, and tailored to reflect which soft constraints the entity is foremost concerned with addressing.

Keywords: Optimization, Scheduling, Mixed Integer Linear Programming

1.   Introduction

The summer conference housing program at UWSP offers housing to participants of a variety of conferences hosted by the university each summer. These include the Special Olympics Wisconsin State Summer Games, the National Wellness Conference, and the American Suzuki Institute, among others. Historically, desk scheduling for the 20+ conference staff members to cover shifts in the ten distinct residence halls had been completed by hand from scratch by the summer conferences coordinator. A new schedule had to be completed each week, tailored to the specific requirements of the incoming camps. The generation of both personal and hall schedules had taken approximately six hours to complete each week. The authors seek to represent this complex, real-world problem as a mathematical model and solve it to optimality. Its success will be measured by a comparison of quantifiable, predetermined preferences, as well as by its reception by the summer conference coordinator and conference staff.

   Scheduling is a common theme in optimization. Job shop scheduling is described as the assignment of resources to tasks in order to minimize the time span of completing all required tasks.[1] This type of scheduling is especially useful in the manufacturing industry, when machine time must be allocated so that materials can be processed in the least amount of time, thereby saving money. Solutions to the nurse scheduling problem have also been achieved using operations research. Miller, et al. summarize this problem as optimally balancing a hospital's individual nurse requests with staffing coverage.[2] Lastly, high school and university timetabling seek to schedule resources at educational institutions. Pillay discusses this group of problems and solution methods in "A survey of school timetabling research".[3]

Numerous variations of these problems exist; the most common focus on assigning teachers to disjoint classes of students and scheduling exams at a university level.

This paper presents a specific scheduling problem based on hourly shifts wherein a secondary group of employees is introduced. These employees are able to work, but should only be scheduled when the primary group of employees is unable to cover all scheduled shifts. An implemented solution is also presented.

2. Solution Method

Of the many considerations that must be taken into account when solving this scheduling problem, the prioritization of constraints on the schedule into requirements and preferences is paramount. The structure for instances of this problem is defined below. These are separated into requirements and preferences.

2.1. schedule requirements

1. There are two types of staff members. *Conference hosts* are the primary desk staff members who have the sole responsibility of staffing the residence hall desks each week. *Conference swing workers* are secondary desk staff members who assist with the coverage of desk hours as necessary throughout each week. There should be as few swing employees working as possible for each schedule period.

2. There must be a specified number of required workers for each hall on each day during each existing time slot. The available staff members must be allocated to cover these shifts.

3. Staff members cannot be assigned to work more than two shifts per day. These shifts cannot overlap.

4. Staff members can work up to ten hours per day.

5. Staff members can work up to 36 hours per week.

6. Staff members can work up to five days per week.

7. Approved individual requests for certain times or days off must be honored.

2.2. schedule preferences

The following are listed in order of importance:

1. Minimize number of scheduled swing workers.

2. Minimize total number of double shifts by individual workers in a day.

3. Distribute hours evenly for all scheduled workers.

2.3. additional option

1. Staff members need at least ten hours off between shifts occurring on consecutive days. For example, a staff member scheduled until 10 p.m. on a Tuesday cannot begin work until 8 a.m. that Wednesday.

2.4. creating a model

The authors modeled each week of the summer conference housing program with a mixed-integer linear program (MILP), a model type that allows two kinds of constraints: hard constraints and soft constraints. Hard constraints are those that the solution is not allowed to violate. All requirements listed as "Schedule Requirements" are hard constraints in the model. In contrast, soft constraints represent additional priorities which incur penalties when violated. The events listed as "Schedule Preferences" are all expressed as soft constraints in the model. For example, if during a particular week of the summer each of the conference hosts were scheduled a notably uneven number of hours,

this event would incur a significant penalty, and the solution would not be ranked as highly as a schedule where each conference host received a relatively even number of hours. These penalties are taken into account in the objective function.

Restricted by both the hard and soft constraints, the MILP contains decision variables that represent whether or not a conference staff member should be scheduled to work at a particular desk during a particular day at a particular time.

An instance of the problem using actual data from the week starting Sunday, June 2, 2013 is included. During this week, the Special Olympics Wisconsin State Summer Games were held on campus, creating the largest demand on UWSP's desk workers for the entire summer. There are 218,611 described decision variables in this instance of the model. This instance is particularly interesting, since all 478 staffed hours occurred between Thursday and Saturday. The MILP solver seeks the combination of variable values that meets the hard constraints and minimizes the penalties incurred by the soft constraints. The option dictating that workers must have ten hours off between shifts on consecutive days was not implemented by the summer conference coordinator, and thus is unused in this instance of the problem. However, it was used judiciously in later scheduling problems to ensure employee satisfaction.

2.5. set definitions

A number of sets are defined for this model. They are as follows:

Table 1: Set Definitions

Set	Definition
W	set of all workers
H	set of all halls requiring workers
D	set of all days in a week
T	set of all times at which a shift may start
V	set of all shift durations (in hours)
V_t	set of valid shift durations for starting time $t \in T$

The set W will be separated further into two mutually exclusive subsets, W_C and W_Z, to represent workers designated as conference hosts and those designated as swing workers, respectively. That is, $W = W_C \cup W_Z$. During the summer, there were regularly 14 conference hosts and 27 available swing workers.

Halls are to be staffed at hourly intervals. Therefore,

$$T = \{6a, 7a, 8a, ..., 10p, 11p\}. \tag{1}$$

Because the model is generated weekly,

$$D = \{Sunday, Monday, ..., Saturday\}. \tag{2}$$

Specific dates are unnecessary.

The set V_t ensures that shifts are paired with a duration that does not violate the specified time constraints. For example, because no hall desk is open past midnight, it does not make sense to allow for an eight hour shift starting at 8 p.m.

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2.6. decision variable

The authors define a binary variable $S(w,h,y,t,d)$ to denote whether or not a shift is worked by a worker $w \in W$ at hall $h \in H$ on day $y \in D$, starting at time $t \in T$ and having a duration of $d \in V_t$ hours.

2.7. constraints

Each worker can have at most two shifts per day. Since double shifts are to be penalized, a binary variable $B(w,y)$ is introduced. It is set to one on day y featuring worker w working two shifts. So,

$$\sum_{\substack{h \in H \\ t \in T \\ d \in V_t}} S(w,h,y,t,d) \leq 1 + B(w,y), \qquad \forall w \in W, \forall y \in D. \tag{3}$$

Because workers are able to have two shifts in one day, it is imperative that these shifts do not overlap. Given a time of day t, let $U_t \subseteq T \times V$ be the set of valid shift starting times and durations that include time t. Then

$$\sum_{\substack{h \in H \\ (a,d) \in U_t}} S(w,h,y,a,d) \leq 1, \qquad \forall w \in W, \forall y \in D, \forall t \in T. \tag{4}$$

As an example, if $S(\text{``}Cai\text{''}, \text{``}MayRoach\text{''}, \text{``}Monday\text{''}, \text{``}8a\text{''}, 8) = 1$, then $S(\text{``}Cai\text{''}, \text{``}Burroughs\text{''}, \text{``}Monday\text{''}, \text{``}3p\text{''}, 6) = 0$. Otherwise, the worker would be scheduled twice between 3pm and 4pm.

The crux of the model is its ability to properly fulfill the staffing requirements stipulated by the summer conference coordinator. Thus, given the amount of workers R_{hyt} required to staff hall h on day y at time t,

$$\sum_{\substack{w \in W \\ (a,d) \in U_t}} S(w,h,y,a,d) = R_{hyt}, \qquad \forall h \in H, \forall y \in D, \forall t \in T. \tag{5}$$

It should be noted that this is a strict equality constraint. Under no circumstances is it acceptable for there to be more than or less than the required amount of workers, R_{hyt}.

The authors now define a binary variable, $K(w,y)$, to determine whether or not a worker w works on day y. This variable will be used to simplify a later constraint which takes into account the number of days worked by a particular worker. The result is

$$\sum_{\substack{h \in H \\ t \in T \\ d \in V_t}} S(w,h,y,t,d) \leq 2 \cdot K(w,y), \qquad \forall w \in W, \forall y \in D. \tag{6}$$

Workers must stay under the maximum number of days of work allowed per week by the summer conference coordinator. M is defined as this number. Therefore,

$$\sum_{y \in D} K(w,y) \leq M, \qquad \forall w \in W. \tag{7}$$

Workers must stay under the maximum number of hours allowed each week by the summer conference coordinator. E denotes this limit, which is applicable to all workers. The authors also define $L(w)$ to be a binary variable used to determine whether or not worker w was scheduled at all for the entire week. This will be taken into account in the objective function as an attempt is made to minimize the number of scheduled conference swing workers. The constraint is given as

$$\sum_{\substack{h \in H \\ y \in D \\ t \in T \\ d \in V_t}} d \cdot S(w,h,y,t,d) \leq L(w) \cdot E, \qquad \forall w \in W. \tag{8}$$

A new continuous variable, X, is defined to be the maximum number of hours given to any single worker. This value of X will be minimized in the objective function in an attempt to spread out the number of hours given to conference hosts.

$$\sum_{\substack{h \in H \\ y \in D \\ t \in T \\ d \in V_t}} d \cdot S(w,h,y,t,d) \leq X, \qquad \forall w \in W. \tag{9}$$

Workers must stay under the maximum number of hours per day, as stipulated by the summer conference coordinator. A will be this daily hour limit. Thus,

$$\sum_{\substack{h \in H \\ t \in T \\ d \in V_t}} d \cdot S(w,h,y,t,d) \leq A, \qquad \forall w \in W, \forall y \in D. \tag{10}$$

Furthermore, individuals are able to ask off for entire days, pending approval of the summer conference coordinator. $G \subseteq W \times D$ is the set of workers requesting off for certain days. This is represented as

$$\sum_{\substack{h \in H \\ t \in T \\ d \in V_t}} S(w,h,y,t,d) = 0, \qquad \forall (w,y) \in G. \tag{11}$$

Finally, approved requests off of work for certain times on certain days must be honored. The authors use (w,y,t) to be an ordered triplet in the set $Q \subseteq W \times D \times T$, where worker w is asking off for time t on day y. These requests are honored with the constraint

$$\sum_{\substack{h \in H \\ (a,d) \in U_t}} S(w,h,y,a,d) = 0, \qquad \forall (w,y,t) \in Q. \tag{12}$$

Although unused in some instances, the following constraint states that ten hours off must be given to workers working consecutive days. Let $(y_1,t_1,d_1) \in D \times T \times V$ and $(y_2,t_2,d_2) \in D \times T \times V$ be day/time/duration combinations that violate this rule. Let h_1, $h_2 \in H$ be two different halls. Then

$$S(w,h_1,y_1,t_1,d_1) + S(w,h_2,y_2,t_2,d_2) \leq 1, \qquad \forall w \in W. \tag{13}$$

This constraint ensures employees have enough rest between shifts, thereby contributing to their job satisfaction.

2.8. objective function

The first priority of the objective function is to minimize the number of scheduled conference swing workers, which are in the set W_Z. If a worker $w \in W$ is scheduled during the week, then binary variable $L(w) = 1$, as explained in (6). It is preferred that conference hosts are the primary desk staff members and they receive as many scheduled desk hours as is feasible.

Secondly, the objective function minimizes the total number of double shifts, denoted as $B(w,y)$ for worker $w \in W$ on day $y \in D$. Ideally, conference workers will only be scheduled to work one shift in a given day.

Finally, the objective function seeks to evenly distribute the hours for all conferences workers each week. When conference swing workers are scheduled, the summer conference coordinator wishes to provide them with the same number of hours as the conference hosts. As described in (7), this variable is defined as X.

The linear objective function for the model is:

$$\text{minimize } 100 \cdot \sum_{w \in W_Z} L(w) \;\; + \;\; 10 \cdot \sum_{\substack{w \in W \\ y \in D}} B(w,y) \;\; + \;\; X. \tag{14}$$

Because of the weights used in the objective function, no hours will ever be taken from conference hosts and given to a swing worker who would not otherwise be scheduled to work. Similarly, workers will not have hours taken away in order to facilitate another double shift.

3. Results

The two schedules will be compared below, starting with the schedule generated by hand for the aforementioned week of June 2, 2013. The summer conference coordinator used a less structured framework from which to build this particular instance of the schedule. Workers were allowed to work more hours each day than the model allows. Furthermore, workers could work more than two shifts on a given day.

Workers with names beginning with a 'C' are the 14 conference hosts. Workers with names beginning with a 'Z' are conference swing workers who received hours for the week. No real worker names are used here. Only 'Chuck' had a request off processed for the model. This individual asked off for Saturday and after 2 p.m. on Friday.

The results of the schedule, as created by hand by the summer conference coordinator, are listed in the following table.

Table 2: Hand-generated Schedule

Worker	Hours	Double Shifts
Cai	21	3
Caitlin	22	3
Caroline	21	4
Carrie	25	3
Cecelia	23	4
Chantelle	26	4
Charlie	24	4
Chuck	17	2
Cindy	24	4
Conley	**27**	6
Connie	24	3
Connor	**27**	4
Crystal	25	4
Curtis	26	4
Zachary	16	2
Zadie	24	2
Zahara	15	1
Zaire	20	2
Zander	12	2
Zane	16	1
Zara	14	1
Zavion	16	3
Zayden	13	1
Total	**478**	**67**

The bold numbers represent the value of X. Note that nine swing workers receive hours, adding 900 to the objective unction's value. The 67 double shifts add another 670, while the X accounts for 27. The objective function value for the model created by the summer conference coordinator is 1597.

Next, the results of the schedule, as optimally calculated through use of the mixed-integer linear program, are listed in the table below.

Table 3: Optimal Schedule

Worker	Hours	Double Shifts
Cai	**24**	1
Caitlin	20	1
Caroline	23	1
Carrie	22	0
Cecelia	23	0
Chantelle	21	0
Charlie	23	1
Chuck	18	0
Cindy	21	1
Conley	**24**	0
Connie	23	0
Connor	23	0
Crystal	**24**	1
Curtis	**24**	0
Zachary	**24**	0
Zadie	**24**	0
Zahara	**24**	0
Zaire	23	1
Zander	23	0
Zane	23	1
Zara	**24**	0
Zavion	0	0
Zayden	0	0
Total	**478**	**8**

Again, the bold numbers above represent the value of X. In the optimal solution, seven swing workers account for 700 of the objective function's value, eight double shifts account for 80, and X accounts for 24. The objective function for the optimal schedule is 804.

Two swing workers from the hand-generated schedule are not given any hours in the optimal schedule. This is desirable, as an optimal schedule will use as few swing workers as possible. Their primary duties do not involve working at residence hall desks. There is no preference given to any particular swing worker when hours must be assigned to them; a schedule where all of one swing worker's hours are given to a swing worker without any hours would retain the same objective value.

4. Conclusion

The dramatic improvement to the objective function between the schedule completed by hand and the schedule created by the mixed-integer linear program illustrates the effectiveness of the model. Furthermore, the summer conference coordinator was able to save several hours each week - scheduling was summarily reduced to the input of weekly parameters. Multiple employees noticed the cleaner, error-free approach to scheduling and commented on its improvement (generally with regard to a sudden, noticeable decrease in the number of double shifts).

The model was used for the majority of the summer after its potency was realized. Its high degree of flexibility allows for it to be applied to situations in which businesses or other institutions have multiple locations to staff, with specific staffing requirements at certain time intervals.

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6. Contact

Eli C. Towle and Kane P. Mach may be contacted at `eli.c.towle@gmail.com` and `kane.mach@gmail.com`, respectively.

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