# Production And Measurement Of Four Degree Of Freedom Photonic States With Correlated Photons 

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#### Abstract

The realization of a practical quantum computer depends on the development of single and multiple qubit logic gates. However, multiple degree of freedom logic gates are difficult to demonstrate experimentally due to the complexity arising from qubit-to-qubit decoherence. Photons seem ideal candidates for qubits due to their low interaction with their environment. A combination of optical elements has been used to produce and manipulate photonic states with multiple degrees of freedom. These manipulations simulate computations with a logic-gate set comprising of photonic states. A four-degree-of freedom system consisting of correlated photon pairs produced by down-conversion, traveling through a Mach Zehnder interferometer, and interacting with wave plates and polarizers was created. The interference pattern of the photons in the interferometer was verified using coincidence-counting electronics. It was observed that when the information carried by these photons remains indistinguishable, the photons interfere with themselves. However, interference disappeared when such information was made distinguishable. Nevertheless, the interference pattern reappeared when a polarizer was placed after the interferometer. The polarizer erased the information carried by the photon's polarization. The results from these experiments demonstrate that photonic states could be used as multiple degree of freedom qubits to perform computations and to possibly scale a quantum computer.


## Keywords: Photonic states, quantum computing, distinguishability

## 1. Introduction

Wires and logic gates in computers have become increasingly small, so small that they will soon reach atomic scale sizes $^{1}$. At such small scale, quantum mechanics plays an important role in the behavior of these computer components. Therefore, a machine that understands the behavior of systems at such small scales is necessary: a quantum computer. In 1982, Nobel Laureate Richard Feynman first proposed the idea of a computer that works under the principles of quantum mechanics ${ }^{1}$. According to Feynman, a quantum computer should be able to process information on quantum bits rather than classical bits. A bit is the fundamental unit of information, representing the distinction between two possible logic states, conventionally called 0 and 1 . A "bit" also refers to a physical system that stores a bit of information ${ }^{1}$. The quantum analog of a classical bit is known as qubit. However, while the state of a classical bit can only be 1 or 0 at the time, a quantum bit can be at both states simultaneously. That is, a quantum bit, or qubit, can be 0 and 1 simultaneously. This phenomenon suggests that a huge speedup in computational power would result from replacing all current computing technology, which stores information in bits, with a quantum computer that stores information in qubits. Similar to classical computations, quantum computations can be broken into a sequence of logic gates that act on a few qubits at the time ${ }^{2}$. Wires are used to carry information around the circuits, while logic gates perform manipulations of the information, converting it from one form to another.

However, the main difference between a classical and a quantum logic gate is the way they manipulate bits. Classical gates manipulate the classical bits, 0 or 1 , one at the time, whereas quantum logic gates can manipulate a system with an arbitrary number of qubits ${ }^{2}$. A qubit can be represented by the state of a quantum mechanical system such as an atom, a photon, or nuclear spin ${ }^{1}$. The quantum phenomenon that describes a system existing at multiple states simultaneously is known as superposition. Superposition is the defining feature of quantum mechanics that allows systems such as electrons or photons to exist in two or more places at once. Hence, qubits can exist in a superposition of 0 and 1 simultaneously ${ }^{1}$. To realize a quantum computer, qubits must retain their quantum properties to perform computations and read out their output. However, physical systems such as atoms and nuclear spins interact with their surroundings, making them susceptible to noise ${ }^{3}$. This type of quantum noise is known as decoherence. Ideally, qubits should remain isolated from their surroundings in some sort of closed system to prevent this inherent type of noise from affecting computations. Achieving a perfectly isolated system, however, has become one of the major challenges in the physical realization of a quantum computer.

An attractive physical system for representing qubits is a photon because photons do not interact very strongly with their surroundings. In fact, they can be guided long distances with negligible losses, as has been observed in various communication systems that use optic fibers ${ }^{3}$. Photons can carry information in their polarization state. Polarization refers to the direction of oscillation of the electric field of a photon. As a result, photonic states, such as polarization, can be used as qubits. The polarization state of a photon can be represented mathematically as a vector. For example, if a photon is vertically or horizontally polarized, its state of polarization can be expressed respectively as shown in equation (1).

$$
\begin{equation*}
|V\rangle=\binom{0}{1} \text { or }|H\rangle=\binom{1}{0} \tag{1}
\end{equation*}
$$

For a quantum bit, the logic state 0 corresponds to a two-dimensional vector, $\binom{1}{0}$, and the logic state 1 corresponds to the vector $\binom{0}{1}$. As a result, the vertical and horizontal polarization states of a photon can represent the logic states 1 and 0 respectively ${ }^{1}$. If a quantum computer were ever built, algorithms would perform computations in parallel. This refers to a computer's capability to carry out many computations simultaneously in superposition ${ }^{4}$. Although, superposition is the quantum mechanical feature that permits two logic states to exist simultaneously, superposition is not the only effect that would be responsible for a computational speedup. Entanglement is another feature that is currently being exploited as quantum computing technology research progresses. Entanglement is a physical phenomenon that manifests itself as a strong correlation among particles, where the state of each particle cannot be described independently; instead, a state is given for the entire system as a whole ${ }^{5}$. Accordingly, multiple-qubit logic gates could be created using the entanglement shared by photons, thus exponentially increasing the speed of computation of a quantum computer.

As a number of other physical systems, photons can exhibit entanglement. Entangled photons can be produced through a process known as spontaneous parametric down-conversion. Down-conversion is a process where a nonlinear crystal is used to split photons into other pairs of photons ${ }^{5}$. The idea behind parametric down-conversion is similar to that of a dispersion prism, where a beam of white light traveling through a dispersion prism splits into its spectral components. When these beams exit the prism, they bend at different angles, and each exiting beam corresponds to a particular wavelength. In accordance with the laws of conservation of energy and momentum, the total energy and momenta of these exiting beams is equal to the energy and momentum of the incident white light beam. Similarly, when photons (pump photons) pass through a down-conversion crystal, they split into pairs, historically known as idler and signal photons ${ }^{6}$. Their combined energy and momenta is equal to the energy and momentum of the pump photons. Although the dispersion of light in a prism is a good way to illustrate how downconversion works, this analogy is not fully correct. Signal and idler photons are correlated in the sense that they have equal polarization states, which in turn are perpendicular to the polarization state of the pump photons. Thus, polarization qubits can be formed by parametric down-conversion.

Photons, and therefore qubits, can be manipulated to process information using several optical elements such as those in a Mach Zehnder interferometer. Figure 1 shows a schematic representation of a Mach Zehnder interferometer and a source of correlated photons. Source S produces pairs of correlated photons - idler and signal photons. The idler photons are sent to an avalanche photodiode detector (APD), while the signal photons are sent through a Mach Zehnder interferometer to another APD detector. One of the mirrors in the interferometer (Mp) moves, making one of the arms longer than the other. The difference in path lengths introduces a phase shift
between the photons travelling through each of the interferometer arms. As a result, an interference pattern will be observed at the detector placed at output $a$ of the interferometer.


Figure 1: Schematic of Mach Zehnder interferometer and source of correlated photons ${ }^{6}$
One way to analytically understand how photons respond to the manipulations resulting from adjusting the components of a Mach Zehnder interferometer is by calculating the probabilities of certain events to occur. For example, the probability of detecting a photon at output $a$ of figure 1 could be obtained by examining the interaction of signal photons with the interferometer components. Useful mathematical tools to represent this type of interactions are probability amplitudes. Unlike classical physics, quantum mechanics does not provide exact quantities of physical variables. Instead, only probabilities of events to occur can be calculated. Probability amplitudes can be represented by the Greek letter phi $\phi$; the modulus squared of this quantity represents the probability of an event to occur, $P=|\phi|^{2}$. Thus, in photon interference experiments, the event where a signal photon arrives at a detector through one of the paths of the Mach Zehnder interferometer can be represented by a probability amplitude $\phi$, and the probability of this event to occur is $P=|\phi|^{2}{ }^{6}$.
Similar to sound waves, water waves, and other types of waves, probability amplitudes follow the principle of superposition. To illustrate these ideas consider the Mach Zehnder interferometer presented in figure 1 where a source of light, $S$, sends two beams of photons to two different detectors. The beam sent through the interferometer has two indistinguishable paths through which the beam could reach the detector at output $a$ : path 1 represented by $l_{1}$, and path 2 represented by $l_{2}$. Accordingly, if source $S$ sends single photons through the interferometer, each photon will choose either path $l_{1}$ or path $l_{2}$. The event whereby a photon arrives to the detector at output $a$ through path $l_{1}$ can be represented by the probability amplitude $\phi_{1}$, and the probability of this event to occur is equal to $P_{1}=\left|\phi_{1}\right|^{2}$. Similarly, if a photon arrives to the same detector by taking path $l_{2}$, the probability of this event to occur is $P_{2}=\left|\phi_{2}\right|^{2}$. Thus, if an experiment is conducted such that the apparatus cannot distinguish the path taken by a photon, the probability amplitude of that photon arriving to the detector at output $a$ is the superposition of the probability amplitudes representing all the possible alternatives to arrive to that detector; that is, $\phi=\phi_{1}+\phi_{2}$. Consequently, the probability of a photon arriving to the detector at output $a$ is $P=\left|\phi_{1}+\phi_{2}\right|^{25}$. Alternatively, if an experiment is capable of determining which path of the interferometer a photon took to arrive to the detector, the probability of such event to occur is equal to the sum of the probabilities of each available path to arrive to that detector. That is, $P=P_{1}+P_{2}=\left|\phi_{1}\right|^{2}+\left|\phi_{2}\right|^{25}$.

One of the optical elements that can be used to simulate computations with correlated photons is a half-wave plate. In this experiment, half-wave plates are used to rotate the polarization of photons by 90 degrees. By placing a halfwave plate in one of the interferometer arms, the polarization of the signal photons taking that path will be changed, thus making such path distinguishable. As a result, the interference pattern should disappear provided that the path taken by the signal photons is no longer indistinguishable. By carefully analyzing this situation, it is observed that the signal photons travelling through the interferometer carry information encoded in their polarization states. Whenever, this information is revealed (or the path is made distinguishable), the photons stop behaving the way they are supposed to, and the expected interference pattern disappears. On the other hand, if the information carried by the photon's polarization states remains "encrypted" (indistinguishable), the photons behave normally and an interference pattern is observed. The gist of this experiment lies in the fact that the information revealed when the path of the signal photons travelling through the interferometer is made distinguishable can be erased to force the photons to behave normally. If a half-wave plate is placed in one of the interferometer arms to change the signal photons' polarization, their polarization can be reverted to its original state by placing a polarizer at the exit of the interferometer. A polarizer is another linear optical device that acts more or less like a filter. A polarizer filters out photons with polarization state parallel to an internal axis, called extinction axis, and transmits photons with polarization state parallel to the orthogonal axis, called the transmission axis ${ }^{7}$. By placing a polarizer in such a way
that the signal photons' polarization state is reverted to its original state, the interference pattern of the signal photons should reappear. As a result, the information about polarization, which resulted from placing a half-wave plate in the interferometer, would have been erased.
Just like the polarization state of a photon can be expressed as a vector, matrix operators acting on the state of this photon can be used to represent half-wave plates and other optical elements. For example, the operator that represents a half-wave plate whose optic axis can be rotated is given by equation (2)

$$
\widehat{W}_{\lambda / 2}(\theta)=\hat{R}(\theta) \widehat{W}_{\lambda / 2} \hat{R}(-\theta)=\left(\begin{array}{cc}
\cos 2 \theta & -\sin 2 \theta  \tag{2}\\
\sin 2 \theta & \cos 2 \theta
\end{array}\right)
$$

where $\theta$ is the angle between the direction of polarization of the photons, and the plate's optic axis. Also, the operator $\hat{R}(\theta)$, which transforms a square basis into a diagonal basis, is defined as

$$
\hat{R}(\theta)=\left(\begin{array}{cc}
\cos \theta & -\sin \theta  \tag{3}\\
\sin \theta & \cos \theta
\end{array}\right)
$$

Similarly, a polarizer with its transmission axis oriented along some arbitrary direction $H$ and rotated by angle $\theta$ is given by equation (4)

$$
\hat{P}_{H}=\left(\begin{array}{cc}
\cos ^{2} \theta & \sin \theta \cos \theta  \tag{4}\\
\sin \theta \cos \theta & \sin ^{2} \theta
\end{array}\right)
$$

Furthermore, the path taken by a signal photon passing through a Mach Zehnder interferometer can be expressed in terms of state vectors, which are often referred to as propagation directions ${ }^{5}$. Because the arms of a Mach Zehnder interferometer are orthogonal to each other, the available directions of propagation in the interferometer can be expressed as vectors, $|X\rangle$ and $|Y\rangle$, as shown in figure 2.


Figure 2: Mach Zehnder interferometer showing directions of propagation. BS: beam splitter, M: mirror ${ }^{5}$
Matrix operators can also represent the optical components that comprise a Mach Zehnder interferometer. The operator for a symmetric, non-polarizing beam splitter and a mirror are given by equations (5) and (6) respectively.

$$
\begin{align*}
\widehat{B} & =\frac{1}{\sqrt{2}}\left(\begin{array}{ll}
1 & i \\
i & 1
\end{array}\right)  \tag{5}\\
\widehat{M} & =\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right) \tag{6}
\end{align*}
$$

In addition, attaching one of the interferometer mirrors to a piezoelectric stack and applying a voltage can cause a phase shift between the interferometer arms; this phase shift is given by equation (7).

$$
\hat{A}=\left(\begin{array}{cc}
e^{i \delta_{1}} & 0  \tag{7}\\
0 & e^{i \delta_{2}}
\end{array}\right)
$$

where $\delta$ represents the phase difference, and the subscripts 1 and 2 refer to the arms of the interferometer. Finally, a single operator representing the entire Mach Zehnder interferometer can be constructed by grouping the terms given by equations (2) through (7) following the order with which the photons traveling thorough the interferometer encounter each optical component.
The expressions given above are a nice way to represent the effect of linear optical elements used in experiments with photons. However, in quantum mechanics, it turns out that making a measurement of the state of a photon does not yield the precise state of such photon, but rather the probability of finding a photon in such state. Moreover, the act of measuring the state of a signal photon in itself constitutes another operation. This operation is the outer product of the photon state vectors, which is also known as projection. Thus, the probability of finding a photon, which entered the interferometer in the state $\left|\psi_{i}\right\rangle$, in the propagation direction state $|X\rangle$ at the output of the interferometer is given by equation (8)

$$
\begin{equation*}
\left.P_{X}=\left|\hat{P}_{X}\right| \psi_{f}\right\rangle\left.\right|^{2} \tag{8}
\end{equation*}
$$

In this experiment, the concept of Hilbert space is also important. A system comprising of signal photons travelling through an interferometer is a four-dimensional Hilbert space: two dimensions for direction of propagation ( $|X\rangle$ and $|Y\rangle$ ) and two dimensions for polarization $(|H\rangle$ and $|V\rangle)$. For example, combinations among these four dimensions can be computed with equation (9)

$$
|X H\rangle=\binom{1}{0} \otimes\binom{1}{0}=\left(\begin{array}{l}
1  \tag{9}\\
0 \\
0 \\
0
\end{array}\right)
$$

where the symbol $\otimes$ indicates an algebraic operation known as the Kronecker product ${ }^{5}$.

## 2. Experimental Method

Figure 3 shows a schematic representation of the setup used in this experiment. Pump photons with approximately 405 nm wavelengths and $15-50 \mathrm{~mW}$ are produced with a GaN diode laser. These photons enter a barium-borate (BBO) crystal, which converts them into pairs of entangled photons through parametric down-conversion. In addition, A HeNe laser is used to align all optical components. The BBO crystal is oriented such that correlated photon pairs leave the crystal at 3 degrees from its central axis. The idler photon of each entangled photon pair is directed towards an avalanche photodiode detector.


Figure 3: Layout for experiments with correlated photons. The interferometer components are beam splitters (BS), metallic mirrors (M), band-pass filters (F), avalanche photodiode detectors (A, B, C) and photon dump (D) ${ }^{5}$

The other photons (signal photons) from each pair enter a Mach Zehnder interferometer, and arrive at another detector placed at the interferometer output. Similarly, one of the interferometer mirrors is mounted on a liner stage where a piezoelectric stack is used to move the mirror and produce a phase shift in the signal photons' path. In addition, a half-wave plate is placed in one of the arms of the interferometer. Because half-wave plates, in addition to changing the polarization state of signal photons, introduce a phase shift in the path of the photons, an additional dummy half-wave plate (with its optic axis perpendicular to the polarization of the signal photons) is placed in the other arm of the interferometer. This dummy half-wave plate corrects for the phase shift caused by the other halfwave plate placed in the interferometer, and equalizes both of the photon paths.

This experiment can be broken into three stages. In the first stage of the experiment, the optic axis of the half-wave plate is set to the vertical direction, which is the same as the polarization state of the photon pairs leaving the downconversion crystal. The dummy half-wave plate remains in the orthogonal direction (horizontal) for all stages of the experiment. In this stage, the path taken by the signal photons travelling through the interferometer arms is indistinguishable. Moreover, in the second stage of the experiment, the half-wave plate is rotated such that its optic axis is at $45^{\circ}$ with respect to the horizontal. This half-wave plate rotates the polarization of the signal photons by $90^{\circ}$; that is, the signal photons' polarization is changed from the vertical to the horizontal direction. In this case, the path taken by the photons traveling the interferometer is distinguishable because the polarizations of the signal photons traveling through either path of the interferometer are orthogonal to each other. Lastly, in the third stage of the experiment, a polarizer with its transmission axis forming a $45^{\circ}$ angle with respect to the horizontal is placed along the $X$-direction of propagation at the output of the interferometer. In this case, the photons passing the polarizer will have the same polarization state (at $45^{\circ}$ from the horizontal), which makes them indistinguishable again. As a result, the path information that was being carried by the signal photons' polarization state was erased along the $X$-direction after the interferometer.

## 3. Mathematical Model

As discussed earlier, this experiment can be expressed as a group of vector and matrix operations as follows. The signal photons entering the Mach Zehnder interferometer have the state $|X V\rangle$, which indicates that these photons enter the interferometer vertically polarized and in the $X$-direction. The Mach Zehnder interferometer can be represented as a single matrix operator that contains the operations performed by each individual component of the interferometer, and containing information about the polarization and propagation direction of the signal photons. The matrix describing the interferometer is therefore given by equation (10)

$$
\begin{equation*}
\hat{Z}_{d p}(\theta)=(\hat{B} \otimes \hat{I})(\hat{A} \otimes \hat{I}) \widehat{W}_{\lambda / 2}(\theta)(\widehat{M} \otimes \hat{I})(\hat{B} \otimes \hat{I}) \tag{10}
\end{equation*}
$$

where the subscripts $d_{p}$ indicate direction of propagation states, and the angle $\theta$ represents the rotation of the halfwave plate's optic axis. Furthermore, $\hat{I}$ is the identity matrix, and $\widehat{W}_{\lambda / 2}(\theta)$ is the matrix that represents a half-wave plate rotated an angle $\theta$, which is given by equation (11)

$$
\widehat{W}_{\lambda / 2}(\theta)=\left(\begin{array}{cccc}
\cos 2 \theta & \sin 2 \theta & 0 & 0  \tag{11}\\
\sin 2 \theta & -\cos 2 \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{array}\right)
$$

The matrix given by equation (11) combines the polarization and propagation direction operations. The upper left and bottom right $2 \times 2$ polarization sub-matrices represent elements along the $X$ and $Y$-directions respectively. These sub-matrices locate the rotatable half-wave plated along the $X$-direction of one arm, and the dummy half-wave plate in the $Y$-direction of the other arm. The off-diagonal sub-matrices are zero because they would mix the polarization components of one direction with those of the other direction, which a wave plate cannot do ${ }^{5}$.

In the first stage of this experiment, the half-wave plate placed in one of the arms of the interferometer has its optic axis parallel to the polarization state of the signal photons (vertical). Therefore, $\theta=0$, and when the initial
state, $|X V\rangle$, of the signal photons is affected by the matrix operator given by equation (10), the output state of these photons is given by equation (12)

$$
\hat{Z}_{d p}(0)|X V\rangle=\frac{i}{2}\left(\begin{array}{c}
0  \tag{12}\\
-e^{i \delta_{1}}-e^{i \delta_{2}} \\
0 \\
e^{i \delta_{1}}-e^{i \delta_{2}}
\end{array}\right)
$$

where $\delta$ is the phase shift produced by the interferometer arms. The terms in the second and fourth rows of equation (12) represent interference ${ }^{5}$. Measuring the final state of the signal photons is equivalent to making a direction-ofpropagation projection (outer product). The probability of finding signal photons at such output is, therefore, given by equation (13)

$$
\begin{equation*}
\left.P(0)=\left|\left(\hat{P}_{X} \otimes \hat{I}\right) \hat{Z}_{d p}(0)\right| X V\right\rangle\left.\right|^{2}=\frac{1}{2}(1+\cos \delta) \tag{13}
\end{equation*}
$$

Similarly, for the second stage of this experiment, the half-wave plate is rotated by $45^{\circ}$ or $\pi / 4$, and the output state of the signal photons leaving the interferometer is given by equation (14)

$$
\hat{Z}_{d p}(\pi / 4)|X V\rangle=\frac{i}{2}\left(\begin{array}{c}
e^{i \delta_{1}}  \tag{14}\\
-e^{i \delta_{2}} \\
-e^{i \delta_{1}} \\
-e^{i \delta_{2}}
\end{array}\right)
$$

Equation (14) does not contain interference terms, which means that no interference pattern will be observed at the output of the interferometer. Furthermore, the probability of detecting a signal photon at the output of the interferometer is given by equation (15)

$$
\begin{equation*}
\left.P(\pi / 4)=\left|\left(\hat{P}_{X} \otimes \hat{I}\right) \hat{Z}_{d p}(\pi / 4)\right| X V\right\rangle\left.\right|^{2}=\frac{1}{2} \tag{15}
\end{equation*}
$$

Hence, there is not interference because the paths of the interferometer are distinguishable, and the information encoded in the polarization of the signal photons is visible. Finally, the third stage of this experiment involves adding a polarizer along the $X$-direction after the interferometer, which is rotated by $45^{\circ}$ from the horizontal. The matrix operator that represents a polarizer for the four-degree-of-freedom photon system is given by equation (16)

$$
\hat{E}=\left(\begin{array}{rlll}
1 / 2 & 1 / 2 & 0 & 0  \tag{16}\\
1 / 2 & 1 / 2 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

where the upper left and lower right sub-matrices of equation (16) correspond to the transmitted polarizations in the $X$ and $Y$-directions respectively. As a result, equation (17) gives the probability of detecting these photons

$$
\begin{equation*}
\left.\left|\left(\hat{P}_{X} \otimes \hat{I}\right) \hat{E} \hat{Z}_{d p}(\pi / 4)\right| X V\right\rangle\left.\right|^{2}=\frac{1}{4}(1-\cos \delta) \tag{17}
\end{equation*}
$$

## 4. Results

The first stage of this experiment consisted of detecting coincidences, allowing the photons to travel through the interferometer. No wave plates or polarizers were placed in the path of the photons or after the interferometer. Figure 4 shows the interference pattern of single photons when the path by the signal photons was indistinguishable. Coincidences were recorded every 10 seconds, and the average number of coincidences in each collection time appear in figure 4.


Figure 4: Interference of single photons with indistinguishable paths
Moreover, figure 5 shows the interference pattern of single photons when one of the paths of the interferometer was made distinguishable.


Figure 5: Interference pattern of single photons with distinguishable paths
Distinguishability was obtained by placing two half-wave plates in the arms of the Mach Zehnder interferometer. One of the half-wave plates was used as a dummy plate to cancel the phase shift introduced by the other half-wave plate. The latter plate was oriented with its optic axis at 45 degrees from the vertical. Lastly, figure 6 shows the interference pattern of photons when a polarizer was placed at the output of the interferometer in addition to the half-wave plates.


Figure 6: Interference of single photons with erased distinguishability

## 5. Discussion and Conclusions

A Mach Zehnder interferometer appears to be ideal to produce and manipulate multiple degree of freedom photonic states. Accordingly, the interference patterns resulting from these manipulations provide evidence of how photons can be used to store and process information. For example, the interference pattern presented in figure 4 results from allowing the photons to travel trough the interferometer without making their paths distinguishable. That is, whether a photon is reflected or transmitted at a particular beam splitter, or how a photon reaches the detector is unknown.

The fact that an interference pattern is observed implies that the information encoded in the polarization state of the photons travelling through the interferometer is encoded when the paths taken by the photons remains indistinguishable.

On the other hand, when the paths taken by the photons in the interferometer is made distinguishable, the information that was previously encoded in the polarization state of the photons is now visible. The pattern presented in figure 5 shows that the photons do not interfere with themselves anymore due to the fact that their path, and therefore polarization state, is now distinguishable. Although making one of the photon paths distinguishable altered the sinusoidal interference pattern observed in figure 4, revealing the information encoded in the polarization state of the photons, the polarizer placed at the output of the interferometer erased this information. The interference pattern presented in figure 6 is evidence that by forcing the photons' polarization to a particular state as determined by the polarizer, the interference pattern reappears. That is, the polarizer erased the information that had previously been uncovered by the half-wave plate.

As a means of comparison, the probability curves suggested by equations 13,15 , and 17 are shown in figure 7 . The red, solid line represents the probability of detecting photons in coincidence when the paths of the interferometer are indistinguishable. On the other hand, the flat, solid, blue line represents the probability of detecting photons when the path they took was made distinguishable. The reduced amplitude green line represents the interference pattern of photons when a polarizer was placed at the output of the interferometer, causing the interference pattern to reappear. Notice that the interference pattern presented in figure 4 resembles the red solid curve in figure 7. Similarly, the data presented in figure 5, where the paths taken by the photons is distinguishable, resembles the blue flat line in figure 7 . Moreover, notice that the probability of detecting coincidences when the photon paths are distinguishable is constant and equal to one half. The data in figure 5 closely follows a constant value as well, which is approximately one half of the maximum amplitude seen in figure 4 . Lastly, the data presented in figure 6 follows a reduced amplitude interference pattern, which resembles the green, reduced amplitude interference pattern observed in figure 7. The reduced amplitude results from the fact that the polarizer placed at the output of the interferometer blocks $50 \%$ of the photons passing through it. Furthermore, notice that the sinusoidal pattern shown in figure 4 is 180 degrees out of phase with the pattern observed in figure 6. A similar trend is observed in figure 7 where both the red and green solid lines appear out of phase by a similar amount. These correlations demonstrate that the probability of detecting photons at the output of the interferometer is proportional to the number of coincidences measured.


Figure 7: Probability of detecting photons in coincidence
The data presented above demonstrates that four-degree-of-freedom photonic states can be produced using linear optical elements, and can be measured using coincidence counting electronics. In addition, these photons can be manipulated, and the photonic states resulting from these manipulations contain information that can be visible or invisible, depending on the type of manipulation. In the case where the path taken by the photons traveling through
the interferometer is indistinguishable, the information encoded in their polarization state is invisible; this result is observed in the non-collapsing interference pattern. On the other hand, when the path taken by the photons traveling through the interferometer is distinguishable, the information encoded in their polarization state is visible, and the interference pattern disappears. Moreover, this visible information can be reverted to its original state by using a polarizer, as indicated by the reappearance of the interference pattern despite of the reduced amplitude. The fact that the experimental data shown above closely follows the trends of the probability curves calculated theoretically demonstrates that the probability of detecting coincidences given a particular setup is proportional to the number of coincidences detected. Lastly, the results from these experiments demonstrate that photonic states could be used as multiple degree of freedom qubits to perform computations provided that information can be encoded in the polarization state of photons.

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