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A Sharp Upper Bound On k-Color Connectivity

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Abstract

How many different types of security do we need to ensure that our network is fully protected? Likewise, how do we effectively optimize a trucking route? While we have vastly different questions, our answer is the same. We apply edge-colorings on graphs to answer this question. Represent each terminal hub with a *vertex*, and each direct link between hubs with an *edge* between the two vertices. We call this collection of vertices and edges a *graph*. For each type of connection security, assign a distinct *color* to the corresponding edges. If we want to transfer information from hub u to hub v through a series of consecutive direct links between intermediate hubs, then we draw a *path* in G. Our question now becomes, "Given a graph G, how many colors do we need for there to exist some *edge-coloring* where, between every two vertices u and v in G there exists a u, v-path using k different colors?" In other words, what is the *k-color connection number* of G, or $cc_k(G)$? We show the k-color connection number for wheel graphs and provide upper bounds for complete bipartite graphs as well as doubly-chorded cycles of small length. In doing so, we improve on the conjecture by Coll et al. that $cc_k(G) \leq 2k - 1$ for all graphs G.

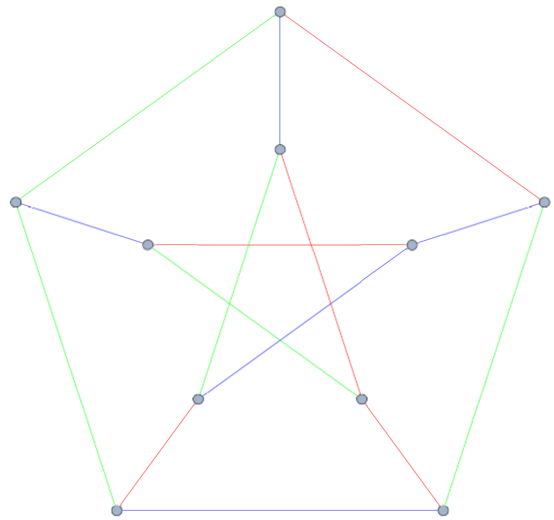
Keywords: Security, Network, Color

1. Introduction

A quick and secure mode of communication is pertinent and necessary in times of crisis. Li and Sun [3], describe how the transfer of sensitive information between separate entities may need to pass through a number of intermediaries. More specifically, consider the following situation. A network consists of roads, each section of which may contain its own security type. Supposing any information passed between two locations must enter at least k different types of security checkpoints, how many different types of security does our network need? Accomplishing the task of securely passing information, made for an intractable problem. Thankfully, a graph theoretic model arose to more adequately address the issue.

A *graph* is a collection of points (*vertices*) that are adjoined by lines (*edges*). Let $V(G)$ and $E(G)$ denote the sets of vertices and edges in a graph G, respectively. Two edges sharing a vertex are *adjacent*, and a collection of adjacent edges and vertices in a graph is called a *path*. A path with an edge between the first and last vertices is a *cycle*. A cycle with n vertices is denoted C_n. The *length* of a path or cycle is its number of edges. An *edge-coloring* on G assigns a color to each edge in G. In 2015, Coll, Luu, Magnant, Martin, and Pyron [2] defined a graph G to be *k-color connected* if between every two vertices $u, v \in V(G)$, there is a u, v-path in G with at least k differently colored edges. The *k-color connection number* of a graph G, denoted $cc_k(G)$, is the minimum number of colors needed to make G k-color connected.

4-edge-colored graph



5-edge-colored graph

The idea of k-color connectivity was inspired by Chatrand, Johns, McKeon and Zhang, who in 2008 [1] defined a *rainbow path* to be a path whose edges all had different colors, and an edge-colored graph to be *rainbow connected* if every pair of vertices contain a rainbow path between them. Additionally, the authors of [1] defined the *rainbow connection number* of a graph G to be the minimum number of colors needed for G to be rainbow colored. Both k-color connectedness and the k-color connection number are similar to their rainbow counterparts, the difference being that a k-color connected graph may have repeated colors on the same path so long as there are enough different edge colors.

Chartrand et al. established the rainbow connection number for (most notably) multipartite, complete, and complete bipartite graphs. Coll et al. found the k-color connection number for complete graphs, and cycles and showed the lack of existence of a k-color connection number for any graph, G, with a bridge, $e \in E(G)$ where $G - e$ is disconnected, for $k > 1$. Coll et al. also found the k-color connection number for wheel graphs, but their coloring needed modification. We present the modified coloring in Section 2. We also provide an upper bound for $cc_k(K_{m,n})$ in the same section.

Our main results are in Section 3, where we improve on the following conjecture by Coll et al.

Conjecture 1 (Coll et al. [2]). For any graph G for which $cc_k(G)$ exists,

$$cc_k(G) \leq 2k - 1.$$

Conjecture 1 would certainly be sharp by assigning each edge of C_{2k-1} a unique color. We improve on Conjecture 1 by determining the k-color connection number for a cycle C with two *chords*, edges outside the C between two vertices in C. Now, for $\ell = 2k - 2t - 2$, define C_ℓ^{**} to be $C_\ell \cup \{1, k - t\} \cup \{t + 2, k + 1\}$. For $\ell = 2k - 2t - 3$, define C_ℓ^{**} as $C_{2k-2t-3} \cup \{1, k - t - 1\} \cup \{t + 2, \left\lceil \frac{2k-t-3}{2} \right\rceil\}$.

Theorem 1 Letting C_ℓ^{**} where $\ell = 2k - 2t - 2$ or $2k - 2t - 3$, $cc_k(C_\ell^{**}) = \ell - 2$.

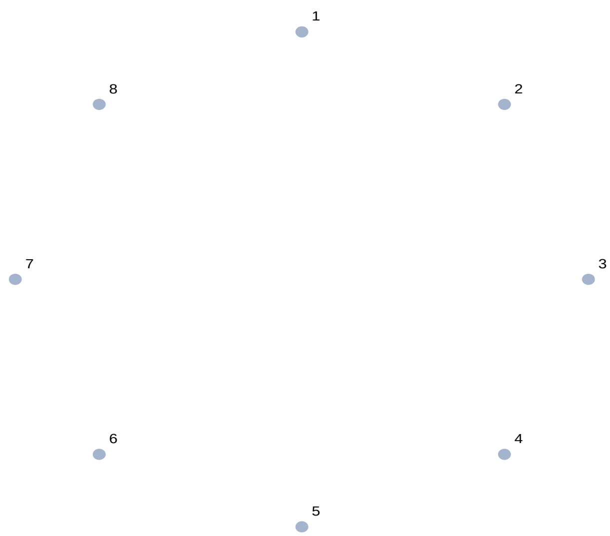
We conclude in Section 4 by showing that to prove Conjecture [2k-1], it suffices to find the k-color connection number for all chorded cycles.

2. Minor Results

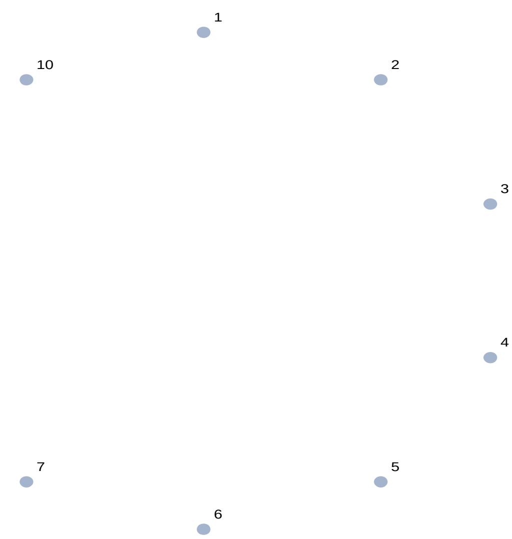
Before presenting our results for chorded cycles, we first state the k-color connectivity of complete bipartite graphs and wheel graphs. A *complete bipartite* graph is a graph consisting of vertex sets A and B such that two vertices are adjacent if and only if one is in A and the other is in B. When $|A| = m$ and $|B| = n$, the complete bipartite graph is denoted $K_{m,n}$.

Theorem 2 For all integers $k \geq 1$, we have $cc_k(K_{\frac{k}{2}+1,\frac{k}{2}+1}) \leq k + 3$.

For $k < 3$ we see C_4 and so $cc_k(C_4) = k$, the number of edges in $K_{\frac{k}{2}+1,\frac{k}{2}+1}$ is less than $2k - 1$, so suppose $k \geq 3$. We can view $K_{\frac{k}{2}+1,\frac{k}{2}+1}$ as a cycle with $\frac{k}{2} - 1$ chords in each of $\frac{k}{2} + 1$ matchings. Use $k + 2$ edge colors to color consecutive cycle edges until all colors have been used. Use one edge color, distinct from those used on the cycle, to color all remaining edges.



6-color connected $K_{4,4}$



8-color connected $K_{5,5}$

Next, we outline the method used to follow a path with k colors.Suppose $|i - j| > 2$, and without loss of generality, let $i = 1$. If j is odd, then let $P_{1,j} = v_1v_2 \dots v_{j-2}v_{k+2}v_{k+1} \dots v_j$. If j is even, then let $P_{1,j} = v_1v_2 \dots v_{j-1}v_{k+2}v_{k+1} \dots v_j$. Now let $|i - j| \leq 2$. Without loss of generality, let $i = 1$. If $j > \frac{k}{2} + 1$, then let $P_{1,j} = v_1v_{k+2} \dots v_j$. If $j < \frac{k}{2} + 1$, then let $P_{1,j} = v_1v_2 \dots v_j$.

Coll et al. showed in [2] that the for a graph G with subgraph H, $cc_k(G) \leq cc_k(H)$. We use their result to generalize Theorem 2 for all complete bipartite graphs.

Theorem 3 ([2]) A 2-connected graph G containing a subgraph H with $cc_k(H) = l$ satisfies $cc_k(G) \leq cc_k(H)$.

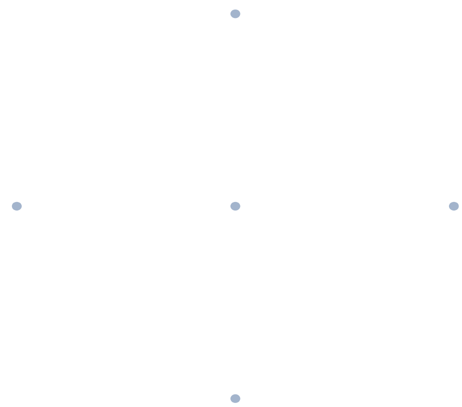
Corollary 4 For all integers $m, n \geq \left\lceil \frac{k}{2} \right\rceil + 1$, we have $cc_k(K_{m,n}) \leq k + 3$.

See Theorem 3.

The *wheel* graph on n vertices, denoted W_n, is defined as $C_{n-1} + \{v_0\}$. Where the union of the vertex v_0 and C_{n-1} results in v_0 being adjacent to all $v \in V(C_{n-1})$.

Theorem 5 For $1 \leq k \leq n - 1$, we have $cc_k(W_n) = k + 1$.

Note that Theorem 5 was stated and proved in [2]; however, the coloring required modification.



4-color connected W_5

Using a vertex labeling $\{v_0, \dots, v_{n-1}\}$, where v_0 is the hub vertex. Color the edges $\{v_1v_2, \dots, v_{n-1}v_1\}$ using distinct colors. Now, for v_iv_j, color v_0v_i using the same color. Without loss of generality, we follow,

$$P_{1,j} = v_1v_2 \dots v_{j-1}v_0v_nv_{n-1} \dots v_j.$$

Necessarily, we must consider ending at the hub vertex, v_0, of W_n,

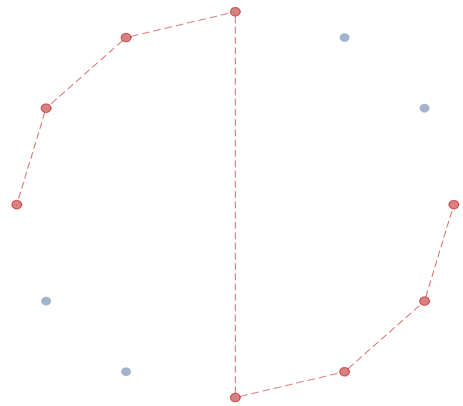
$$P_{1,0} = v_1v_2 \dots v_nv_0.$$

3. Main Results

Before stating our main results, we define a few necessary terms and state an immediate but helpful fact.

Two pairs of vertices (u, v) and (x, y) on a cycle C are *parallel* if there exists a consecutive labeling of the vertices on C such that $u \leq x < y \leq v$. Two chords uv and xy are *parallel* if their corresponding pairs (u, v) and (x, y) are parallel. Likewise, (u, v) and (x, y) are *intersecting* if they are not parallel. Additionally, two chords uv and xy are *intersecting* if (u, v) and (x, y) are intersecting.

Fact 1 A parallel chord to the pair (u, v) does not increase the length of the longest u, v-path.



C_n with diameter chord

We now prove several results for even and odd doubly-chorded cycles. For the rest of the paper, let $\ell = 2k - 2t - 2$ with $t \geq 1$ and $k \geq 3 + 5t$, or let $\ell = 2k - 2t - 3$ with $t \geq 1$ and $k \geq 6 + 5t$.

A *diameter chord* on a cycle of length ℓ is a chord between vertices u and $u + \left\lfloor \frac{\ell}{2} \right\rfloor$ (mod ℓ). We first prove that every doubly-chorded cycle of length ℓ contains a diameter chord. We then show that precisely one doubly-chorded cycle of length ℓ (up to symmetry) has a path of length at least k between all pairs of vertices. Note that this condition is necessary, but not sufficient, for k-color connectedness. We then state our main result, that $cc_k(C_\ell^{**}) = \ell$.

Lemma 1 The chords of a k-color connected doubly-chorded ℓ-cycle are both diameter chords.

For ease of notation, we refer to vertices v_i by their label i. Consider a single-chorded ℓ-cycle C^{**} with a non-diameter chord. Without loss of generality, let one chord be an edge from 1 to χ with $\chi \in \{3, \ldots, \left\lfloor \frac{\ell}{2} \right\rfloor\}$, while the second may be in any location such that the two chords are intersecting. We claim that for every vertex $i \in \{\chi + 1, \ldots, \ell\}$, there is some vertex v_j in C^{**} such that C^{**} contains no v_i, v_j-path of length at least k. By Fact 1, there are no paths of length at least k between v_1 and any vertex in $\{\left\lfloor \frac{\ell}{2} \right\rfloor - t + 1, \ldots, k\}$; analogously, there are no paths of length at least k between vertex χ and any vertex in $\{\left\lfloor \frac{\ell}{2} \right\rfloor + 1 - t + \chi - 1, \ldots, k + \chi - 1\}$. (If $k + \chi - 1 > \ell$, then subtract ℓ.) Also by Fact 1, each vertex $i \in \{\chi, \ldots, \left\lfloor \frac{\ell}{2} \right\rfloor - t\}$ contains no path of length at least k to vertex $i + \left\lfloor \frac{\ell}{2} \right\rfloor$ (mod ℓ); analogously, each vertex $j \in \{k + \chi - 1, \ldots, \ell\}$ contains no path of length at least k to vertex $j + \left\lfloor \frac{\ell}{2} \right\rfloor$ (mod ℓ). Hence, we only need to show that for each vertex in $S = \{k + 1, \ldots, \left\lfloor \frac{\ell}{2} \right\rfloor + 1 - t + \chi - 2\}$, there is some vertex to which there is no path of length at least k. Note that for $S \neq \emptyset$ if and only if $k + 1 \leq \left\lfloor \frac{\ell}{2} \right\rfloor + 1 - t + \chi - 2$, or $\chi \geq 2t + 3$.

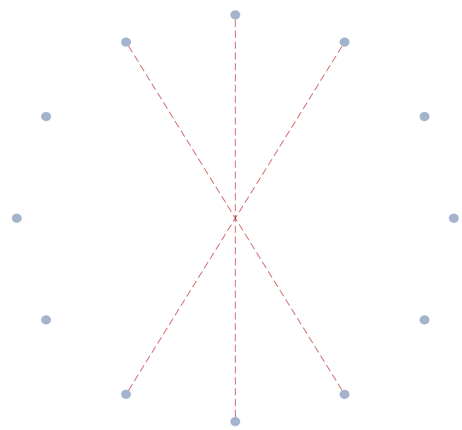
We show that each $i \in S$ such that $i < \left\lfloor \frac{\ell}{2} \right\rfloor + \frac{\chi+1}{2}$ has no path of length at least k to the vertex $i + (k - 1) - \ell$. Note that there are only two paths from vertex i to vertex $i + (k - 1) - \ell$ that use the chord 1χ. Those two paths are $P_1 = \{i, i + 1, \ldots, \ell, 1, \chi, \chi - 1, \ldots, i - k + 2t + 1\}$ and $P_2 = \{i, i - 1, \ldots, \chi, 1, \ldots, i + (k - 1) - \ell\}$. The path P_1 has length $(\ell + 1 - i) + 1 + \chi - (i + (k - 1) - \ell) = 2\ell - k - 2i + \chi + 1$. Now, since $k + 1 \leq i < \left\lfloor \frac{\ell}{2} \right\rfloor + \frac{\chi+1}{2}$, we have $\left\lfloor \frac{\ell}{2} \right\rfloor + 2 + t < \ell + \left\lfloor \frac{\ell}{2} \right\rfloor + 1 - t - 2i + \chi + 1 < k - 4t + \chi - 3 < k$. Similarly, we see P_2 has length $i - \chi + 1 + (i + (k - 1) - \ell) - 1 = 2i - \ell + k - \chi - 1 < \ell + \chi + 1 - \ell + k - \chi - 1 \leq k$. So all vertices $i \in S$ have no path of length at least k to the vertex $i + (k - 1) - \ell$.

Next, we show that each $j \in S$ such that $j > \left\lfloor \frac{\ell}{2} \right\rfloor + \frac{\chi+1}{2}$ has no path of length at least k to the vertex $j - (k - 1)$. As before, there are only two paths $P_3 = \{j, j + 1, \ldots, \ell, 1, \chi, \chi - 1, \ldots, j - (k - 1)\}$ and $P_4 = \{j, j - 1, \ldots, \chi, 1, \ldots, j - (k - 1)\}$. The path P_3 has length $(\ell + 1 - j) + 1 + \chi - j + k - 1 = \ell + k - 2j + \chi - 1 < \ell + k - (\ell + 1 + \chi) +$

$\chi - 1 = k$; similarly, the path P_4 has length $(j-\chi)+1+(j-k+1)-1 = 2j-k-\chi+1 < (\ell+1+\chi)-k-\chi+1 < k$.

Lastly, we consider the case when χ is odd and the vertex $m = \ell + \frac{\chi+1}{2}$ exists. There is no path of length at least k from m to $m-(k-t-1) = \frac{\chi+1}{2}$. Neither the path $P_5 = \{m, m+1, \ldots, \ell, 1, \chi, \chi-1, \ldots, \frac{\chi+1}{2}\}$ nor $P_6 = \{m, m-1, \ldots, \chi, 1, \ldots, \frac{m+1}{2}\}$ has length k. For P_5 has length $\ell + 1 - m + 1 + \chi - \frac{\chi+1}{2} = \left\lfloor\frac{\ell}{2}\right\rfloor + 1 - \chi < k$, and P_6 has length $m - \chi + 1 + \frac{m+1}{2} - 1 = \frac{m-1}{2} - \chi < k$.

Since every vertex in $\{\chi+1, \ldots, \ell\}$ lacks a long enough path to some vertex in C^*, no additional chord to any of these vertices can lengthen every path. Any other chord included in C^* would be parallel to the pair $(1, k-1)$, which by Fact 1 would still not have a path of length at least k. Hence, C^{**} is not k-color connected.



A C_{12} where dashed lines show vertices without a path of length k

Now we consider the doubly chorded cycle $C^{**}_{2k-2t-2}$ defined as $C_{2k-2t-2} \cup \{1, k-t\} \cup \{t+2, k+1\}$.

Lemma 2 C^{**}_{ℓ} contains a path of length at least k between all pairs of vertices.

We show there is always a v_i, v_j-path in G of length at least k in $C^{**}_{\ell} = C_\ell \cup \{1, \left\lfloor\frac{\ell}{2}\right\rfloor + 1\} \cup \{t+2, k+1\}$. Notice that for v_i in C_ℓ where $i = \{1, \ldots, \ell\}$, the vertices under scrutiny are all v_j where $j = \{i + \left\lfloor\frac{\ell}{2}\right\rfloor + 2t - 1, \ldots, i + \left\lfloor\frac{\ell}{2}\right\rfloor - 1\} \bmod \ell$, as v_i clearly has a path of length at least k to all other vertices. The two chords separate the cycle into quadrants, two large and two small. Without loss of generality, we examine three cases, due to the symmetry of the C_ℓ, namely, when a vertex is incident on a chord, when a vertex is contained in a quadrant of length $t+2$, and when a vertex is contained in a quadrant of length $\left\lfloor\frac{\ell}{2}\right\rfloor - t - 1$.

C_ℓ *where* $k = 8$ *and* $t = 1$

$v_i = v_1$

Let P be a longest v_i, v_j-path in C_ℓ; hence, we have

$$P = v_1, v_\ell, \dots, v_{k+1}, v_{t+2}, v_{t+3}, \dots, v_{\left\lfloor \frac{\ell}{2} \right\rfloor + 1}.$$

Observe that $|P| = \ell - t - 1 \geq k$.

Let $i \in \{\ell + 4, \dots, k + 2\} \mathrm{mod} \ell$ and $j \in \{i + \left\lfloor \frac{\ell}{2} \right\rfloor + 2t - 1, \dots, i + \left\lfloor \frac{\ell}{2} \right\rfloor - 1\} \mathrm{mod} \ell$.

Let P_1, P_2 be two longest $v_i - v_j$-paths in C_ℓ^{**},

$$P_1 = v_\ell, v_1, \dots, v_{k+1}, v_{t+2}, v_{t+3}, \dots, v_{\left\lfloor \frac{\ell}{2} \right\rfloor + 1 - t}, \dots, v_{\left\lfloor \frac{\ell}{2} \right\rfloor};$$

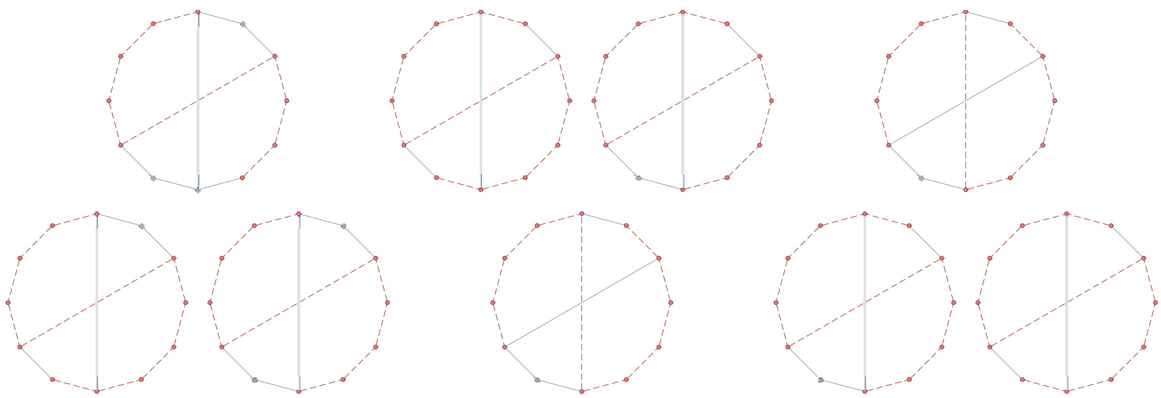
$$P_2 = v_\ell, v_1, \dots, v_{k+1}, v_{t+2}, v_{t+3}, \dots, v_{\left\lfloor \frac{\ell}{2} \right\rfloor + 1}.$$

Observe that the length of P_1 is $\ell - 1 \geq k + 3t$, while the length of P_2 is $\ell - 2t - 2 \geq k + t - 1$, which is greater than or equal to k as a result of $t \geq 1$.
Let $i \in \{2, \dots, t + 1\} \mathrm{mod} \ell$ and $j \in \{i + \left\lfloor \frac{\ell}{2} \right\rfloor + 2t - 1, \dots, i + \left\lfloor \frac{\ell}{2} \right\rfloor - 1\} \mathrm{mod} \ell$
Again, we choose two longest paths, P_1, P_2 in C_ℓ^{**} such that,

$$P_1 = v_{t+1}, v_{t+2}, \dots, v_{k-t}, v_1, v_\ell, \dots, v_{\left\lfloor \frac{\ell}{2} \right\rfloor + 2};$$

$$P_2 = v_{t+1}, \dots, v_1, v_\ell \dots, v_{k+1}, v_{t+2}, v_{t+3}, \dots, v_{\left\lfloor \frac{\ell}{2} \right\rfloor + 1}.$$

The length of P_1 is $\ell - t - 1 \geq k + 2t$, which is certainly greater than k. Next, we can see that the length of P_2 is $\frac{3}{2}\ell - 2t - 1 \geq 2k - 1$, which again is larger than k.



Case 1 for $k = 8, t = 1$ *Case 2 for* $k = 8, t = 1$ *Case 3 for* $k = 8, t = 1$

Lemma 3 C_ℓ^{**} is the *only* k-color connected doubly-chorded cycle.

Let $t \geq 1$, and first consider C_ℓ^{**}. By Lemma 5, we only need to show that no other doubly-chorded cycles besides C_ℓ^{**} contain a path of length at least k between all pairs of vertices. We attempt to construct such a doubly-chorded cycle C^{**}. By Fact 2, we know that C^{**} must contain a diameter chord between (without loss of generality) vertices 1 and $\left\lfloor \frac{\ell}{2} \right\rfloor + 1$. We now show that the other chord must be a diameter chord between $t + 2$ and $k + 1$ (or, equivalently, between $\left\lfloor \frac{\ell}{2} \right\rfloor - t$ and $\ell - t - 1$), or else there is some pair of vertices without a path of length at least k between them. By Fact 1, vertex 1 has no path of length at least k to any vertex in $\{\left\lfloor \frac{\ell}{2} \right\rfloor + 1 - t, \dots, k\}$; likewise, vertex $\left\lfloor \frac{\ell}{2} \right\rfloor + 1$ has no path of length at least k to any vertex in $\{\ell - t + 1, \dots, t + 1\}$. Since the diameter chord from 1 to $\left\lfloor \frac{\ell}{2} \right\rfloor + 1$ only increases the length of a maximum path starting at $\left\lfloor \frac{\ell}{2} \right\rfloor + 2$ or $\frac{3\ell+4}{4}$ by at most 1, by Fact 1, we also have that vertex $\left\lfloor \frac{\ell}{2} \right\rfloor + 2$ has no path of length at least k to any vertex in $\left\{\frac{3\ell-2t+8}{4}, \dots, \frac{3\ell-4t}{4}\right\}$; likewise, vertex $\frac{3\ell+4}{4}$ has no path of length at least k to any vertex in $\left\{\frac{\ell-4t+8}{4}, \dots, \frac{\ell+4t}{4}\right\}$.

Since $k \geq 5t + 3$, we have $\frac{3\ell-4t+8}{4} \geq k + 2$ and $\frac{k-4t+8}{4} \geq t + 3$. The only chords that are not parallel to all vertex pairs without a long enough path are the chords between $(t + 2, k + 1)$ (or, equivalently, between $(\left\lfloor \frac{\ell}{2} \right\rfloor - t, \ell - t)$).

Theorem 1 $cc_k(C_\ell^{**}) = \ell - 2$.

Assign a unique color to the edges $\{v_1 v_2, \dots, v_\ell v_1\}$. Next, for $\{v_1, v_{\left\lfloor \frac{\ell}{2} \right\rfloor + 1}\}$ assign the same color as $\{v_{t+1}, v_{t+2}\}$, and for $\{v_{t+2}, v_{k+1}\}$ use the color from $\{v_{\left\lfloor \frac{\ell}{2} \right\rfloor + 2t}, v_{k+1}\}$. Observe that any u, v adjacent on the cycle have a path of length $\ell - 1$ between them, which has a path using $\ell - 1$ colors. Without loss of generality, we consider the same paths as those used in Lemma 5. For all path lengths considered, notice that subtracting each by 2 represents the fewest number of colors possible on each, due to only 2 edge colors being reused. This is only problematic for P_2 in Case 2, which was specified as $P_2 = v_\ell, v_1, \dots, v_{k+1}, v_{t+2}, v_{t+3}, \dots, v_{\left\lfloor \frac{\ell}{2} \right\rfloor + 1}$, and had length $\left\lfloor \frac{\ell}{2} \right\rfloor + 2$. The length of P_2 means that we could have $\left\lfloor \frac{\ell}{2} \right\rfloor + 2t - 2$ different colors. However, because P_2 uses only one chord, and neither of the two sections of length $t + 1$, it cannot reuse a color. This means that P_2 must have a path with $\left\lfloor \frac{\ell}{2} \right\rfloor + 2t$ colors.

4. Conclusion

We conclude by showing that Conjecture 1 can be proved if one knows the k-color connection number for all chorded cycles. We first state three results by Coll et al. from [2]. Before we do so, a graph is *2-connected* if for every pair of vertices in G, there is a cycle containing both.

Theorem 6 Let K_n denote the complete graph on n vertices.

1. For $k \geq 3$ and $n \geq k + 1$, we have $cc_k(K_n) = k$.

2. For $n \geq 3$ and $k > 1$, we have

$$cc_k(C_n) = \begin{cases} DNE & if\ n \leq 2k - 2, \\ 2k - 1 & if\ n = 2k - 1, and \\ k & if\ n \geq 2k. \end{cases}$$

3. A 2-connected graph G containing a subgraph H with $cc_k(H) = l$ satisfies $cc_k(G) \leq cc_k(H)$.

Item 3 shows that the k-color connection number of G depends only on the subgraphs of G. We now state our result.

Theorem 7 Given k, let $k + 1 \leq n \leq 2k - 2$. Let C' be a cycle on n vertices that includes $0 \leq \chi \leq n - 3$ chords. If for all k and all C' we have $cc_k(C') \leq 2k - 1$, then Conjecture 1 holds.

By Theorem 6, the k-color connectivity of a graph is bounded above by the k-color connectivity of any of its subgraphs. As Coll et al. note, if G is not 2-connected, then $cc_k(G)$ does not exist for $k > 1$, as there is some pair of vertices in G whose only path between them is an edge. Hence, we only need to consider the k-color connection number of 2-connected subgraphs of G.

Since every 2-connected graph contains a cycle C as a subgraph, if $cc_k(C)$ exists, then we have $cc_k(G) \leq cc_k(C) \leq 2k - 1$ by Theorem 6. However, if $cc_k(C)$ does not exist, then consider the subgraph consisting of C and all of its chords. Call this chorded cycle C'. If $cc_k(C') \leq 2k - 1$, then $cc_k(G) \leq cc_k(C') \leq 2k - 1$.

We are currently working toward finding the k-color connection number of triply-chorded cycles. This result may be difficult to prove, however, as it appears the best-located chords are not diameter chords.

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