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Optimal Scheduling for CK Advertisements on TV Programs

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Abstract

Advertising agencies (buyers) place television advertisements orders with the networks (sellers) for their desired commercial slots which will expose their products to a particular target audience. Typically, an advertiser needs information on a certain amount of impressions (i.e., views) for each advertising campaign. This project applies two ways, impression based approach and demographic approach, in evaluation of the advertisements orders for a specific company with modified data provided by Clypd. The results show that the impression-based method is more stable while the demographic-based method is more sensitive to different types of customers, prime-time slots, and non-prime time slots. In comparison, the advertising companies should choose the demographic-based method for more accurate results to maximize the profits for both the advertising companies and the networks.

Key Words: Advertisements, Orders, Sellers, Impressions, Impression-based approach, Demographic-based Approach

1. Introduction

Advertising has been a common practice and has played a crucial role for marketing consumer goods for centuries. Mass distribution of advertisements through newspapers was first made possible by the printing press, but it was the advent of the radio and the television in the twentieth century that ultimately revolutionized advertising by allowing companies to transmit marketing messages into millions of homes around the world simultaneously [1, 2]. Today, television advertisement is a common and efficient way for advertising agencies to expose their products to a specific group of audience. Advertising has become an over $500-trillion-dollar global industry, and although advertising through digital media is growing rapidly, television remains the primary advertising medium with total television advertising expenditures making up approximately 40% of the worldwide total [3].

Advertising agencies assist their clients in placing their television advertisements which will expose their products to a particular target audience. The agencies will analyze all networks for factors such as who watch, when they watch, what is their gender, how old are they, what is their annual income. Typically, an advertiser would like a certain amount of impressions (i.e., views) for each advertising campaign, either for the target audience or the total audience.

These impressions are further categorized as total impressions (number of times the ad was seen) and unique impressions (i.e., the number of different people who saw the ad) [5].

Advertising agencies, such as Clypd, place orders with the networks, such as ABC, for desired commercial slots. A slot is a designated time on a particular network (e.g., ABC on October 1st, 2016, from 10:11–10:14 am) which can run one or more commercials in pods. (For instance, the slot described above could contain six 30-second pods.) The advertising agencies need to choose the proper method of advertising to get their desired sale rate. For example, the agencies may not place their car advertisements on Animal Planet or Discovery and expect the best outcomes in sales. The question is: how do both networks and advertising companies maximize their profits from these commercial advertisements? The advertising agencies need to provide valuable demographic information on the commercial slots so that each buyer can best promote their products. Figure 1, which is provided by Clypd at the 32nd Mathematical Problem in Industry workshop [5], demonstrates the basic schematic diagram between advertising agencies (Clypd), the networks (sellers) and the advertising companies (buyers).

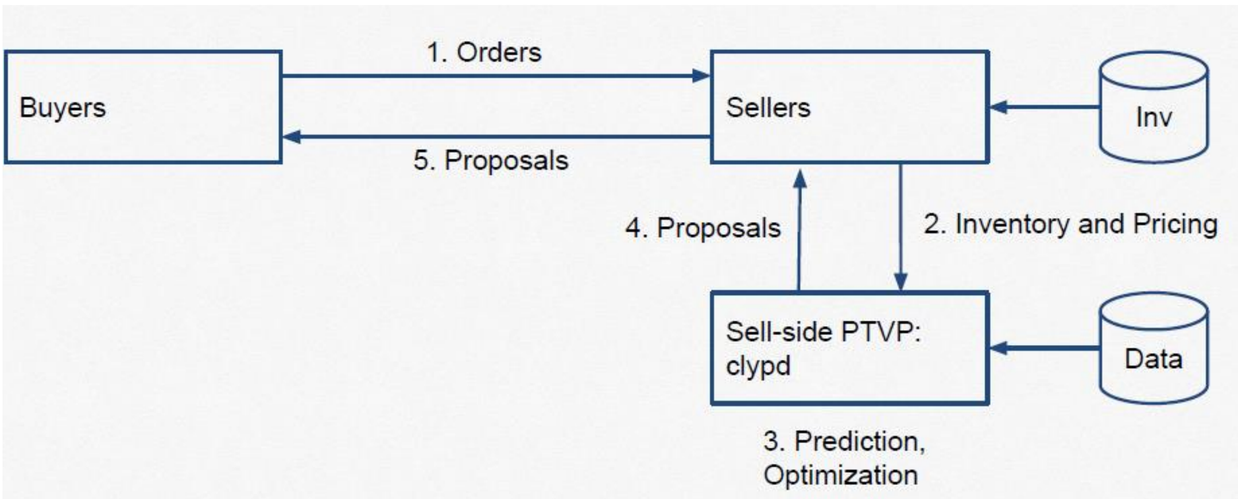

Figure 1: Schematic Diagram between buyers and sellers.

There are generally two types of orders: the constrained orders and the bid orders. The constrained orders require a certain number of impressions for a particular target audience. Then the seller is responsible to assign the commercials with specific time slots based on the historical data provided to the target audience [2]. The bid orders are the orders that the buyers will request specific slots including time, slot and network according to their own marketing analysis. Typically, bid orders will pay a premium fee for their slots due to their selectivity. The remaining slots are then assigned to the constrained orders while matching the target audience characteristics. In an ideal world, the desired slots of two sellers would never overlap. In reality, this happens so often that buyers need a way to determine which company wins and what happens next for the bidder which did not win. Rather, they must make a counteroffer for an incomplete order to maximize their profits. The counteroffer may fall into one of the following two cases: (1) an offer to run a subset of the slots requested, at a reduced rate. Not only is the rate reduced by the number of unfilled slots, but an additional discount is often taken since buyers are typically willing to pay a premium for a specific series of slots; (2) replacement of the unfulfilled slots with other slots, which the seller can hopefully justify as having the same sort of target audience as the unfilled slots. The default option is to at least provide replacements, with or without discounts. In addition, typically the largest advertisers are given priority on receiving complete bids.

In this paper, we will only consider a simple example that the agencies submit a constrained order which is a list of requirements to the networks based on their ‘real requirements’, when they want to advertise their new products. In section 2, we will formulate the models with two different approaches to illustrate this traditional way of ordering. The results will be discussed in section 3 and a conclusion is made in section 4.

2. Model Formulating

The luxury brand, Calvin Klein (CK), wants to showcase its new necktie product to the market. The sale department of CK places an order to the networks with the following requirements: 6 months long campaign; 30 million of male impressions; total $1,000,000 spending; and 5 national channels at evening and weekends. These requirements are based on some ‘real requirements’: at least 5 million of unique impression of males working in private company at the age of 22-45 with yearly salary $150,000 or more; with at least 3 different national channels exposures. Based on these requirements, it is entirely the networks’ responsibility to arrange the commercial slots for CK by utilizing the historical data. To maximize their own profits, the networks tend to accept as many orders as possible though it may sacrifice some of the interests of their customers. However, there is a more efficient method to satisfy both sides of business. When advertising agencies place orders, they conduct market research and predict the best time slots for advertising. Then they will submit a *bid order* with a list of specific time slots.

To illustrate this, Calvin Klein does a targeted market analysis based on its requirements, and then lists the slots they want: (1) 1 May, 2017 on ABC at 20:00; (2) 1 May, 2017 on FOX at 20:03; (3) 1 May, 2017 on CW at 20:05; …; (3714) 30 November, 2017 on ABC at 22:00 with a budget of $850,000.

Although different bid orders are submitted at different times, networks gather the orders before a certain deadline and after then, they begin to distribute the slots for their customers. For that reason, it is more reasonable for us to assume that the bid orders are submitted at the same time. Moreover, a conflict arises when several bid orders request the same slot. Under this circumstance, if the number of bid orders for the same slot is larger than the number of *pods* of the slot, which poses the question on how to distribute the slots to ensure that the networks can maximize their benefits.

Given the restrictions of the constrained orders, we will only focus on the bid orders to provide the sellers a better method of selling. In this paper, we will formulate a mathematical model to compare the two different approaches based on impressions and demographic information respectively, in order to identify a more efficient method of maximizing seller revenues.

Considering that the sellers will not want to lose customers, it is reasonable for sellers to evaluate the price of each individual slot in different orders instead of regarding each order as a whole. Then sellers are able to make a comparison amongst them to decide which one should be accepted. In this way, customers are usually provided a subset of their demands as a counteroffer. Moreover, there exists a *reserved price*, which represents the lowest price that the sellers can accept. Thus, a bid order will not be accepted if valued lower than the reserved price.

The viewer survey gathers information of the impressions based on the age range and gender. In this way, the total number of impressions for a certain slot can be calculated as:

$$
\begin{aligned}
&(total\ number\ of\ impressions\ of\ a\ certain\ slot) \\
&\qquad = \sum (number\ of\ impressions\ for\ every\ age\ bracket\ and\ gender\ indicator)
\end{aligned}
$$

This information can be represented by an ordered pair (a, b), where *a* represents the age range and *b* represents the gender (Table 2.1). For example, the ordered pair (7,2) represents a female in the 31-35 age range.

Table 2.1. Age Range and Gender Values

A	Age Range	a	Age Range	b	Gender
1	0-5	8	36-40	1	Male
2	6-10	9	41-45	2	Female
3	11-15	10	46-50		
4	16-20	11	51-55		
5	21-25	12	56-60		
6	26-30	13	61-65		
7	31-35	14	66+		

In the process of this evaluation, we do not take the effect of reserve price in consideration. This is because if the advertising companies are rejected for the orders lower than the reserve price, then the sellers will not gain any profits due to unfilled slots. The sellers cannot gain profits from the orders valued below the reserve price, but if the reserve price is set to 0, indicating that the reserve price has no effect on rejections, then the number of unfilled slots can be reduced. For the convenience of calculations, each slot is assumed to have only one pod. As for counteroffers, bidders are not awarded slots that they have not initially requested.

2.1 Impressions-Based Approach

In this approach, the value of each slot is based on the impressions surveyed in the market. Each impression is assumed to be of equal value and the total value of a bid order is the sum of each individual slot value. The value p_{ij} to a certain order *i* for slot *j* is based on the total number of impressions for each desired slot:

$$\left(value\ \left(p_{ij}\right) to\ a\ certain\ order\ (i)\ for\ certian\ slot\ (j)\right) = (total\ bid\ i) * (value\ per\ impression) * (total\ number\ of\ the\ impressions\ on\ slot\ j)$$

Where the weight of the slot could be calculated in this way:

$$(value\ per\ impression) = \frac{(bid\ price\ for\ i)}{(number\ of\ desired\ impressions)}$$

Next, we will use the modified data for Calvin Klein to illustrate the impressions-based approach.

Table 2.2. Number of Impressions

Interpretation	**Slot**								
(a, b)	A	B	C	D	E	F	G	H	I
(1, 1)	5	7	0	62	2	0	9	1	9
(2, 1)	24	22	2	53	5	1	14	12	10
(3, 1)	42	39	9	41	9	14	11	18	14
(4, 1)	37	41	15	37	20	9	19	13	1
(5, 1)	21	18	22	21	37	11	27	24	2
(6, 1)	14	19	19	14	49	10	28	36	0
(7, 1)	20	27	14	10	56	17	22	33	4
(8, 1)	9	13	11	5	55	24	17	47	3
(9, 1)	17	21	34	6	50	31	10	44	10
(10, 1)	12	16	39	1	48	17	8	44	14
(11, 1)	11	17	47	0	51	12	3	39	9
(12, 1)	24	14	50	0	37	8	7	35	21
(13, 1)	27	31	56	0	33	10	12	19	11
(14, 1)	33	24	43	1	42	14	1	25	19
(15, 1)	26	11	66	0	29	6	11	38	22
(1, 2)	2	8	1	50	0	1	7	6	5
(2, 2)	21	21	0	44	0	0	2	17	1
(3, 2)	38	29	0	68	1	0	11	15	9
(4, 2)	47	44	9	30	3	7	14	27	17
(5, 2)	26	38	19	21	7	16	20	28	31
(6, 2)	18	24	21	13	19	20	31	21	28
(7, 2)	19	21	14	7	21	34	17	34	33
(8, 2)	9	14	28	1	20	39	11	45	55
(9, 2)	18	20	30	10	14	47	4	55	49

(10, 2)	20	20	49	2	15	68	17	40	68
(11, 2)	19	11	41	0	7	72	17	34	51
(12, 2)	21	30	50	0	9	69	5	32	42
(13, 2)	22	10	55	1	11	55	0	17	39
(14, 2)	35	13	61	0	7	64	1	26	29
(15, 2)	28	19	57	2	3	66	9	33	34
Total Number	665	642	862	500	660	742	365	858	640

Table 2.3. The Requirements of the Orders

Order	Slot									Price
	A	B	C	D	E	F	G	H	I	
Order1	1	0	0	0	0	1	0	1	0	$2,076
Order2	0	0	1	1	0	0	1	0	0	$1,762
Order3	0	1	0	1	0	0	0	0	0	$1,020
Order4	0	0	0	0	1	0	0	1	1	$2,490
Order5	0	0	1	0	0	0	0	0	1	$1,000
Order6	1	1	0	0	1	0	0	0	0	$1,288
Order7	0	0	0	0	0	1	1	0	0	$1,071
Order8	0	1	1	0	1	0	0	1	0	$2,901
Order9	0	0	1	1	1	0	0	0	1	$2,777

In Table 2.3, the values 0 and 1 indicate undesired and desired slots, respectively. For instance, Order1 assigns the value 1 to slots A, F, and H—the only slots the order finds desirable.

In order to determine which bid should be accepted, the value of each slot must first be calculated to ensure that profits are maximized. To do so, it is necessary to calculate the total number of impressions that an order desires. Example Order1 is used to illustrate this process.

Using the data presented in Table 2.3, Order1 requests three slots: A, F, and H. From Table 2.2, the total number of impressions for each slot is added together, thereby calculating the total desired number of impressions for Order1:

$$(total\ number\ of\ Order1\ desired) = 665 + 742 + 858 = 2265$$

Next, the value of a single impression of Order1 is calculated by dividing the total bid price by the total number of impressions:

$$(value\ of\ single\ impression) = \frac{2076}{2265} \doteq 0.917$$

The value of a single impression is then used to find the individual value of each slot, both desired and undesired. Said values are obtained by multiplying the single impression value by the respective slot impression value. The values of each slot for Order1:

$$(value\ of\ slot\ A\ for\ Order1) = 0.917 * 665 \doteq 609.51$$
$$(value\ of\ slot\ B\ for\ Order1) = 0.917 * 642 \doteq 588.43$$
$$\dots$$
$$(value\ of\ slot\ I\ for\ Order1) = 0.917 * 640 \doteq 586.60$$

Table 2.4. Value of Each Slot of a Certain Order

Order	Slot								
	A	B	C	D	E	F	G	H	I
Order1	609.51	588.43	790.07	458.28	604.93	680.08	334.54	786.41	586.60
Order2	678.48	655.01	879.47	510.13	673.38	757.04	372.40	875.39	652.97
Order3	593.96	573.42	769.91	446.58	589.49	662.73	326.01	766.34	571.63
Order4	767.31	740.77	994.62	576.92	761.54	856.15	421.15	990.00	738.46
Order5	442.74	427.43	573.90	332.89	439.41	494.01	243.01	571.24	426.10
Order6	435.44	420.38	564.44	327.40	432.17	485.86	239.00	561.82	419.07
Order7	643.37	621.12	833.97	483.74	638.54	717.87	353.13	830.10	619.19
Order8	638.37	616.29	827.49	479.98	633.57	712.29	350.39	823.65	614.37
Order9	693.73	669.73	899.24	521.60	688.51	774.05	380.77	895.07	667.65

Table 2.4 lists the value of each slot *j* for each order *i*. By doing so, it is easy to compare the value of each slot and to determine which bid should be accepted. However, assigning a value to initially undesired slots is meaningless—it is more reasonable to set the price as 0, indicating the slot was not requested.

Table 2.5. Value of Desired Slots

Order	Slot								
	A	B	C	D	E	F	G	H	I
Order1	609.51	0	0	0	0	680.08	0	786.41	0
Order2	0	0	879.47	510.13	0	0	372.40	0	0
Order3	0	573.42	0	446.58	0	0	0	0	0
Order4	0	0	0	0	761.54	0	0	990.00	738.46
Order5	0	0	573.90	0	0	0	0	0	426.10
Order6	435.44	420.38	0	0	432.17	0	0	0	0
Order7	0	0	0	0	0	717.87	353.13	0	0
Order8	0	616.29	827.49	0	633.57	0	0	823.65	0
Order9	0	0	899.24	521.60	688.51	0	0	0	667.65
Maximum	609.51	616.29	899.24	521.60	761.54	717.87	372.40	990.00	738.46
Accept	Order1	Order8	Order9	Order9	Order4	Order7	Order2	Order4	Order4

Table 2.5 provides a value for a desired slot, but assigns 0 to unwanted slots. The maximum value of each slot is identified, and the respective order is accepted. For example, two bid orders desired slot A (Order1 and Order6). However, the value of slot A as calculated for Order1 is much greater than that of Order6 for the same slot. Thus, Order1 is accepted.

The impression-based method is an effective way to distribute the slots and may lead to increased profit, but it has some limitations. The disadvantage of the impression-based method is that it is not sensitive to viewer surveys, age range and gender indicators. Due to this insensitivity, the value difference between prime time slots and non-prime time slots becomes insignificant. This potentially drives the price of the prime time slot down and the non-prime time slot up. The model cannot effectively differentiate between the slot types, therefore buyers cannot avoid purchasing the non-prime time slot. Furthermore, if the buyers wish to target a particular audience, the impression-based model may not accurately reflect slots that will be viewed by said target audience. To combat the sensitivities of the impression-based model, a more efficient method is posed—the demographic-based approach.

2.2 Demographic-Based Approach

This approach is more complicated, though realistic. The sensitivity of the weight of the slot is improved so that a more accurate value of each slot is obtained. In the demographic-based approach, the impressions from various demographics are valued differently. In that way, we have the value of the slot of a certain order:

$$(value\ of\ slot\ j\ of\ a\ certain\ order) = \sum_{(a,b)} [value\ of\ (a,b)\ of\ slot\ j\ of\ a\ certain\ order]$$

The value of (a, b) of the slot j is defined as:

$$\begin{aligned}&[value\ of\ (a,b)\ of\ slot\ j\ of\ a\ certain\ order]\\ &\qquad = [weight\ of\ (a,b)\ of\ slot\ j\ of\ a\ certain\ order] * (total\ price\ of\ OrderM)\end{aligned}$$

$$\begin{aligned}&[weight\ of\ (a,b)\ of\ slot\ j\ of\ a\ certain\ order]\\ &\qquad = [weight\ of\ (a,b)\ of\ a\ certain\ order] * [impressions\ of\ (a,b)\ of\ slot\ j]\end{aligned}$$

$$[weight\ of\ (a,b)\ of\ a\ certain\ order] = \frac{\sum_j impression(a,b)\ of\ slot\ j}{\sum_{(a,b)}[\sum_j impression(a,b)\ of\ slot\ j]^2}$$

Where *a* represents the age range, *b* represents the gender indicator and *j* represents the slots that the order requires. Note that when calculating the weight, it is only necessary to add the number of impressions that the order desired. The Calvin Klein example is also used here to illustrate the method.

Table 3.1. Weight of the Slots of Each Order

Order	Slot								
	A	B	C	D	E	F	G	H	I
Order1	0.26	0.24	0.38	0.12	0.25	0.38	0.13	0.36	0.30
Order2	0.38	0.36	0.49	0.31	0.34	0.40	0.21	0.45	0.35
Order3	0.46	0.46	0.34	0.54	0.30	0.30	0.24	0.43	0.30
Order4	0.24	0.24	0.39	0.10	0.31	0.35	0.14	0.39	0.30
Order5	0.31	0.28	0.57	0.09	0.30	0.51	0.15	0.45	0.43
Order6	0.33	0.32	0.40	0.20	0.35	0.31	0.18	0.40	0.26
Order7	0.42	0.40	0.67	0.18	0.40	0.75	0.25	0.62	0.58
Order8	0.19	0.19	0.30	0.08	0.24	0.24	0.11	0.28	0.20
Order9	0.24	0.22	0.35	0.14	0.25	0.30	0.13	0.32	0.26

Table 3.2 Value of (a, b) of Order1

Interpretation (a, b)	**Slot**								
	A	B	C	D	E	F	G	H	I
(1, 1)	0.31	0.43	0.00	3.84	0.12	0.00	0.56	0.06	0.56
(2, 1)	9.17	8.40	0.76	20.24	1.91	0.38	5.35	4.58	3.82
(3, 1)	32.08	29.79	6.87	31.32	6.87	10.69	8.40	13.75	10.69
(4, 1)	22.53	24.97	9.14	22.53	12.18	5.48	11.57	7.92	0.61
(5, 1)	12.14	10.40	12.72	12.14	21.39	6.36	15.61	13.87	1.16
(6, 1)	8.67	11.77	11.77	8.67	30.35	6.19	17.34	22.30	0.00
(7, 1)	14.45	19.51	10.12	7.23	40.46	12.28	15.90	23.84	2.89
(8, 1)	7.43	10.74	9.08	4.13	45.42	19.82	14.04	38.81	2.48
(9, 1)	16.14	19.94	32.29	5.70	47.48	29.44	9.50	41.78	9.50

(10, 1)	9.04	12.06	29.39	0.75	36.17	12.81	6.03	33.15	10.55
(11, 1)	7.04	10.88	30.08	0.00	32.64	7.68	1.92	24.96	5.76
(12, 1)	16.60	9.68	34.58	0.00	25.59	5.53	4.84	24.21	14.52
(13, 1)	15.61	17.92	32.37	0.00	19.08	5.78	6.94	10.98	6.36
(14, 1)	24.53	17.84	31.96	0.74	31.21	10.40	0.74	18.58	14.12
(15, 1)	18.79	7.95	47.69	0.00	20.95	4.34	7.95	27.46	15.90
(1, 2)	0.19	0.74	0.09	4.64	0.00	0.09	0.65	0.56	0.46
(2, 2)	8.24	8.24	0.00	17.26	0.00	0.00	0.78	6.67	0.39
(3, 2)	20.79	15.87	0.00	37.20	0.55	0.00	6.02	8.21	4.92
(4, 2)	39.30	36.79	7.52	25.08	2.51	5.85	11.71	22.57	14.21
(5, 2)	18.79	27.46	13.73	15.17	5.06	11.56	14.45	20.23	22.40
(6, 2)	10.96	14.62	12.79	7.92	11.57	12.18	18.88	12.79	17.05
(7, 2)	17.06	18.86	12.57	6.29	18.86	30.53	15.27	30.53	29.63
(8, 2)	8.64	13.44	26.88	0.96	19.20	37.44	10.56	43.20	52.80
(9, 2)	22.30	24.77	37.16	12.39	17.34	58.22	4.95	68.13	60.69
(10, 2)	26.42	26.42	64.74	2.64	19.82	89.84	22.46	52.85	89.84
(11, 2)	24.52	14.19	52.90	0.00	9.03	92.90	21.93	43.87	65.80
(12, 2)	26.45	37.78	62.97	0.00	11.33	86.89	6.30	40.30	52.89
(13, 2)	21.35	9.70	53.37	0.97	10.67	53.37	0.00	16.49	37.84
(14, 2)	45.16	16.77	78.71	0.00	9.03	82.58	1.29	33.55	37.42
(15, 2)	36.71	24.91	74.72	2.62	3.93	86.52	11.80	43.26	44.57
SUM	541.38	502.83	796.95	250.44	510.73	785.16	273.72	749.46	629.85

In Table 3.2, the sum of the values of slots A, F and H are compared to the total price that Order1 bid, as an indication of accurate calculations. By continuing the demographic-based approached, Table 3.3 was constructed, listing slot values of Order1 to Order9.

Table 3.3. The Value of Slots found using the Demographic-Based Model

Order	Slot								
	A	B	C	D	E	F	G	H	I
Order1	541.38	502.83	796.95	250.44	510.73	785.16	273.72	749.46	629.85
Order2	672.80	635.73	855.94	538.00	605.16	705.84	368.06	800.82	616.72
Order3	467.09	471.38	343.28	548.62	305.96	301.34	244.78	435.02	301.27
Order4	603.10	590.57	976.70	253.65	777.08	866.11	356.33	976.67	736.25
Order5	310.35	275.28	565.47	85.50	296.13	512.54	147.31	453.05	434.53
Order6	420.58	411.84	514.12	258.04	455.58	396.42	225.46	520.27	333.86
Order7	452.57	428.40	718.45	190.74	428.76	801.14	269.86	664.19	618.01
Order8	558.30	538.36	863.67	246.30	685.36	691.04	305.47	813.61	569.07
Order9	657.04	621.10	972.35	399.55	696.93	821.81	352.80	901.03	708.17

As aforementioned, it is not necessary to assign values to slots that were not initially requested. The value 0 is assigned to all undesired slots.

Table 3.4. Value of Desired Slots found using the Demographic Model

Order	Slot								
	A	B	C	D	E	F	G	H	I
Order1	541.38	0	0	0	0	785.16	0	749.46	0
Order2	0	0	855.94	538.00	0	0	368.06	0	0
Order3	0	471.38	0	548.62	0	0	0	0	0
Order4	0	0	0	0	777.08	0	0	976.67	736.25
Order5	0	0	565.47	0	0	0	0	0	434.53

Order6	420.58	411.84	0	0	455.58	0	0	0	0
Order7	0	0	0	0	0	801.14	269.86	0	0
Order8	0	538.36	863.67	0	685.36	0	0	813.61	0
Order9	0	0	972.35	399.55	696.93	0	0	0	708.17
Maximum	541.38	538.36	972.35	548.62	777.08	801.14	368.06	976.67	736.25
Accept	Order1	Order8	Order9	Order3	Order4	Order7	Order2	Order4	Order4

The maximum and accept rows are included in Table 3.4, indicating that the order containing the maximum value of a slot is accepted. Doing so maximizes the profits of the seller. By comparing the two methods, it is evident that the results differ slightly. According to the demographics-based approach, slot D should be distributed to Order3 while the impression-based approach indicates it should be distributed to Order9. This is likely due to the audience of the TV program, which the impression-based approach does not consider. From the survey of viewers, it is known that the majority of the audience of slot D are children. When comparing the orders, Order9 requests slots C, D, E, and I while Order3 requests only slots B and D. When analyzing the viewer impressions of each of these requested slots, it is evident that slots B and D have a viewer audience consisting of mostly children, while C, E, and I were not. Therefore, to some extent, it is reasonable to assume that Order3 had listed children as its target audience, thus valuing slot D more than Order9 had. The demographic-based model is more sensitive towards audience type than the impression-based model, which explains the discrepancies of the model results.

3. Results

The stability analysis of both approaches are conducted. A normally distributive perturbation with standard deviation 5% of the original value is added to the prices of the orders. This means that if p_i represents the price of bid order *i*, then it yields the perturbed bid data:

$$p_i \mapsto p_i (1 + X), \quad X \sim N(0, (0.05p_i)^2),$$

where *i* is the natural numbers from 1 to 9. Here, the values of the order are randomly assigned following a normal distribution with standard variation 0.05. The equation of relative error is defined as:

$$(relative\ error) = \frac{(inferred\ value) - (true\ value)}{(true\ value)}$$

Of the calculated relative errors, the largest value is used to indicate the maximum error. By calculations, the relative errors for the impression-based approach and demographic-based approach are 7.77% and 10.32%, respectively. This indicates that the impression-based approach is more stable than the demographic-based approach. However, said relative errors calculated may not precisely represent the stability due to the limited number of orders in this experiment. Furthermore, each bid order may only need to request a subset of slots, which is not considered in the above statistical analysis. However, by implementing statistical analysis that only includes the desired slots, the relative errors of the impression-based approach and demographic-based approach are 7.49% and 9.05%, respectively. Although the impression-based method is still considered to be more stable, the newly found values may closely represent the real values.

4. Conclusion

In summary, we test two different kinds of approaches for the networks (sellers) to distribute the commercial slots to the advertising companies (buyers). The impression-based approach sets the price for each slot by valuing a single impression while demographic-based approach determines the price using the weight of the slots. By comparison of these two approaches, the demographic-based approach is proved to be more sensitive to the impressions of the demographic information including age range, annual salary, and gender of the targeted audience. Furthermore, the demographic-based approach isolates the prime slots and normal slots more easily than the impression-based approach does. Although impression-based model is more stable, by using the demographic-based method, more realistic results can be obtained and the slots are distributed in a way that compliments target audiences. It is suggested that networks (sellers) should use the demographic-based method to evaluate the slots of a bid order and then accept/reject orders based on the evaluation. By doing so, the seller is able to maximize profit while also ensuring that the buyers are able to advertise to their target audiences. For future directions, there are additional factors that will be taken into consideration. For instance, when the network makes a counteroffer to the buyers, they often do so *with replacement*. Counteroffers with replacement may lead to a loss of profit for sellers, while not completely satisfying buyers. In addition to counteroffers, the effect of the reserve price will also be added for more realistic results. With the reserve price being set to 0, seller's profits are decreased by accepting all the bid orders.

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