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Measuring the Granular Density of Modes in 3D

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Abstract

Sand covering the earth, snow on a mountainside, and even plastic balls in a ball pit are all considered granular materials. These athermal materials, while ubiquitous, behave in a unique manner. For instance, granular materials have the ability to behave like all three phases of matter; sand in a sandstorm behaves like a gas, whereas sand flowing in an hourglass behaves like a liquid, and packed sand on a beach behaves like a solid. The solid/liquid transition is known as the jamming transition. This study will experimentally measure the granular density of modes, which is analogous to the density of states in a thermal system. In order to measure the granular density of modes, we need to mimic the randomized motion of thermal particles. We accomplish this using a frequency sweep to vibrate the particles of the granular material allowing us to measure the density of modes using methods from thermal physics. From our measurement of the granular density of modes, we are able to study the jamming transition in a 3D granular material; since, as the jamming transition is approached, there is an excess number of low frequency modes in the density of modes.

Keywords: Granular Materials, Jamming, Density of States

1. Introduction

From the cereal you eat for breakfast, to the sand on the ground you walk upon, granular materials are everywhere. More specifically, granular materials are composed of macroscopic particles that interact strictly through repulsion between their contacts[1-3].

Despite granular materials being so commonplace, a lot is unknown about them. Research in granular physics has many useful applications, such as how granular materials would affect space exploration[4], or investigating how pressure builds in a grain silo[5]. Another important area of granular physics research is granular acoustics. Granular acoustics is an area of interest because the unique way that forces propagate through granular materials makes it difficult to predict how sound will travel through the granular medium[6,7]. The applications of granular acoustics are numerable, including developing sonar-like technology that can detect things underground like oil or land mines, without invasive and often destructive probing.

Of particular interest to this study is the jamming transition which is a phase transition between solid-like and liquid-like behavior in a granular material[8,9]. The jamming transition is one of several phase transitions a granular material can experience. For example, granular materials have the ability to behave like the three basic states of matter[2]; sand can be solid in a sandcastle, flow like a liquid in an hourglass, or act like a gas in a sandstorm. The jamming transition is when the material transitions from the solid state to the liquid state and vice versa. For instance, putting loose sand under high pressure will jam it, then relieving that pressure will unjam it.

Previous studies have found that the jamming transition is accompanied by an excess of low frequency modes in the density of states[10,12], and so the density of states provides a means of probing a granular material's proximity to the

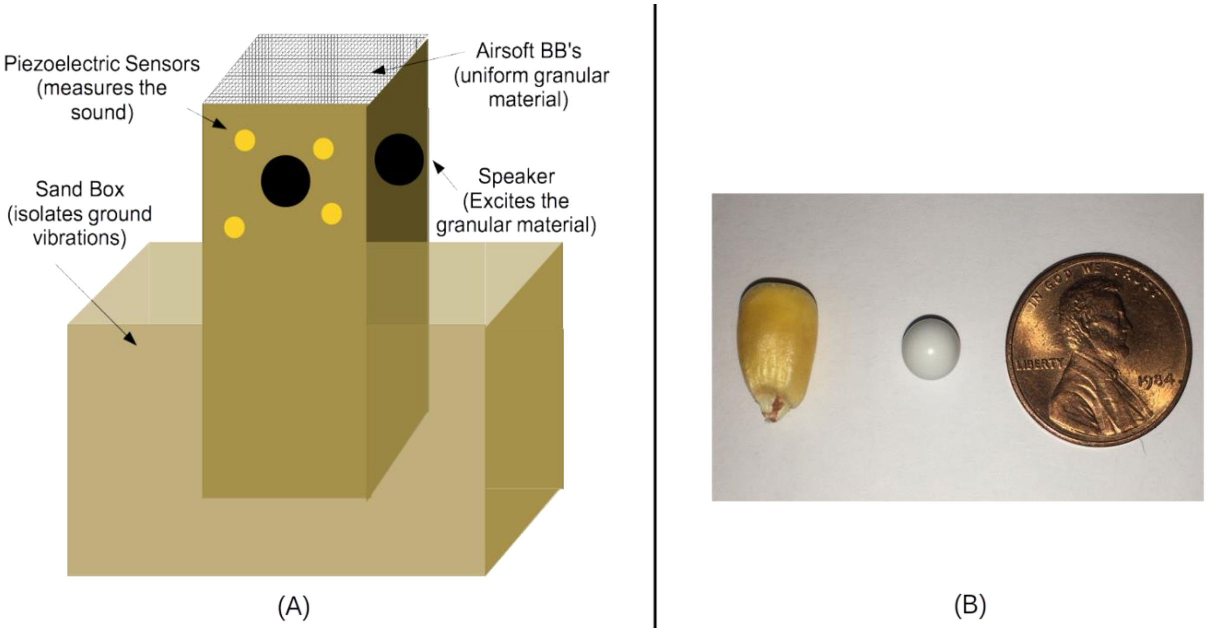

Figure 1:The experimental setup. (A) A schematic of the experimental setup. (B) Examples of the corn and Airsoft BBs used for the granular material along with a penny for comparison.

jamming transition. However, since granular materials are an athermal system and the density of states is a thermal metric, special procedures must be followed in order to measure the density of states[10-12], or density of modes as it is sometimes referred to for granular systems.

This study proposes a method for experimentally measuring the granular density of modes in a 3D material with the intent of comparing a uniform, spherical granular media with a non-spherical granular media. The experiment uses sound to mimic the randomized vibrations that thermal materials experience and thus thermalize the granular material. Then, borrowing from thermal physics, the density of modes can be found using the spectrum of the velocity autocorrelation function of the particles[10,11]. This work will provide a valuable technique for investigating the jamming transition in real 3D granular materials.

2. Experimental Methods

The experimental setup consists of a 31x31x20 cm box that is filled with the granular material to be studied (see Figure 1A). This box is nested inside another box that is filled with sand in order to isolate the setup from ground vibrations coming from the floor of the building. For these experiments, the following two different granular materials are studied: dried corn and Airsoft BB's (see Figure 1B). The Airsoft BB's used in the experiments are the MetalTac Competition Grade BB’s and have a diameter of 5.95±0.01mm and a mass of 0.2 grams. The Airsoft BB’s where chosen due to their uniform spherical shape while corn was chosen due to its non-spherical shape. Using both spherical and non-spherical granular materials allows for comparison between the two density of modes and provides a check on whether or not we are in fact measuring features of the granular material. In particular, due to the corn’s non-spherical shape, corn is less likely to pack into an ordered crystalline structure compared to the spherical airsoft BBs which naturally arrange themselves into a more ordered configuration.

Since granular materials are athermal but the density of states is a property of thermal systems, the granular material must be “thermalized”. The thermalization of the granular material is accomplished acoustically via four speakers attached to each side of the experimental setup (see Figure 1A). Frequency sweeps of 0.01-10 kHz are generated and sent to the speakers by the BK Precision 4052 function generator. Frequency sweeps are used in

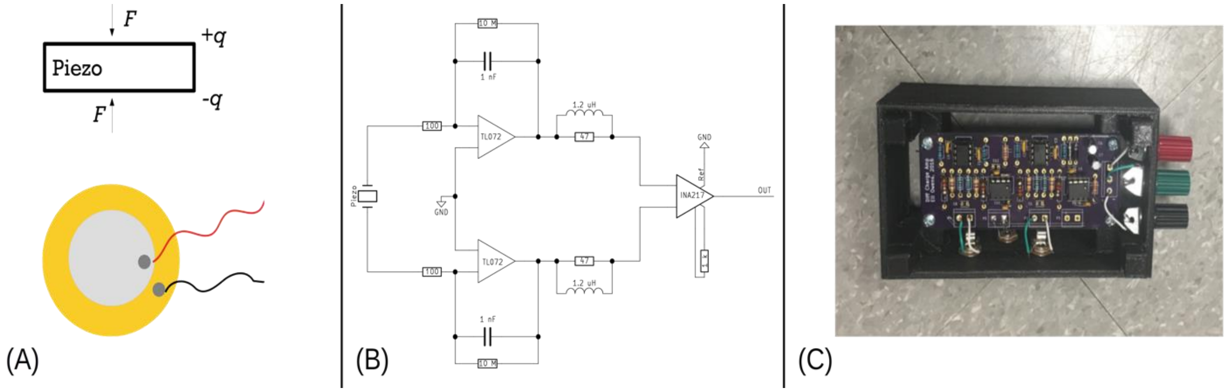

Figure 2: The various pieces of the piezoelectric sensor system. (A) A schematic of the piezoelectric sensor and how they work. (B) A circuit diagram of our signal conditioning circuit. (C) A photo of the finished circuit which is housed in a custom 3D printed box.

order to not favor any one particular frequency. The speakers chosen for this experiment are the CUI INC GF0668, and were chosen because of their approximately flat frequency response in the range of 0.3-20 kHz.

Once the granular material has been thermalized, the response of the material is recorded by five piezoelectric sensors buried in the granular material. An additional sensor is placed near, but outside, the granular material and is used to measure any stray electrical or acoustic interference. The piezoelectric sensors used are the PUI Audio Inc. AB2065B and consist of a 15 mm diameter crystal which when compressed produces a charge output proportional to the strain on the crystal[13] (see Figure 2A). The charge output of each side of the piezoelectric is converted into a voltage via a charge amplifier, then, a differential amplifier subtracts the voltage of one side of the crystal from the other side in order to remove parasitic common mode noise. The circuit that performs this operation is of our own design and construction (see Figure 2B). Finally, this differential signal is fed into a Measurement Computing USB-1608FS-Plus DAQ card where the data is saved to a computer.

Since one of the goals of this experiment is to investigate the jamming transition, we collected data under two different confining pressures. In the first case, the top of our system was left unconfined. While in the second case, four 2 kg weights were placed on top of the system. This produces approximately 800 Pa of confining pressure and compacts the system (moves the system away from the jamming transition).

Using this equipment, a typical data collection run would be carried out as follows: First, the granular material is agitated by hand to rearrange the grains and put the system in a different state from the previous run; the piezoelectric sensors are also placed in the material at approximately the same depth and in an “x” pattern. Second, a frequency sweep is sent from the function generator to the speakers, effectively thermalizing the granular material. Third, the vibrations of the grains are recorded by the buried piezoelectric sensors and saved to the computer for future analysis. Finally, the weights are added to the top of the system to move it further from the jamming transition, and another frequency sweep is sent from the function generator and recorded by the piezoelectric sensors.

In order to make a meaningful measurement of the granular density of modes, many runs must be taken. The above procedure is repeated 20 times for each pressure and grain type. This gives us a final data set consisting of 20 different packings for corn under two different pressure and 20 packings for the Airsoft BB’s under two different pressures.

3. Results and Analysis

Once all the data has been collected, the first step in our data analysis is to clean the data by removing extraneous frequencies such as the ambient 60 Hz noise in the room. This cleaning is done by utilizing the piezoelectric sensor that is placed near, but outside, the granular material. This sensor is under the same ambient conditions as the sensors in the material, but will not respond to the frequency sweep sent in by the function generator and is therefore used to differentiate electrical noise from our acoustic signal. The signal from this ambient sensor is scanned for peaks in frequency space and the first 10 peaks are removed from all the sensors. Many of these peaks are multiples of 60 Hz.

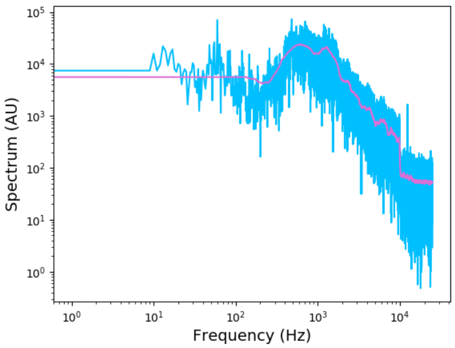

Figure 3: The average spectrum of the piezoelectric sensors. The blue curve has been smoothed to produce the magenta curve.

Next, we need to ensure that the measurements are reflective of the granular material and not of extraneous resonances caused by the system itself (i.e. the box, speakers, sensors, etc.). In order to accomplish this the average spectrum of the five buried sensors is computed using all 80 data runs (20 for each of the two pressures and two grain types). This average is then smoothed to create an overall correction factor which is applied to each of the sensors before computing the density of modes (see Figure 3). This protocol for dealing with system resonances differs from the past experimental procedures[10], in that, the speakers are not corrected to inject a flat power spectrum. However, this novel method of correcting the signal in post processing has the advantage of greater experimental ease and simplicity.

Once the signal from each of the piezoelectric sensors is properly cleaned and corrected, the density of modes is computed using the spectrum of the velocity autocorrelation function. Using this technique, the density of modes is computed for both the corn and the Airsoft BB's at each of the two pressures (see Figure 4). All of this analysis is carried out using Python[14-18].

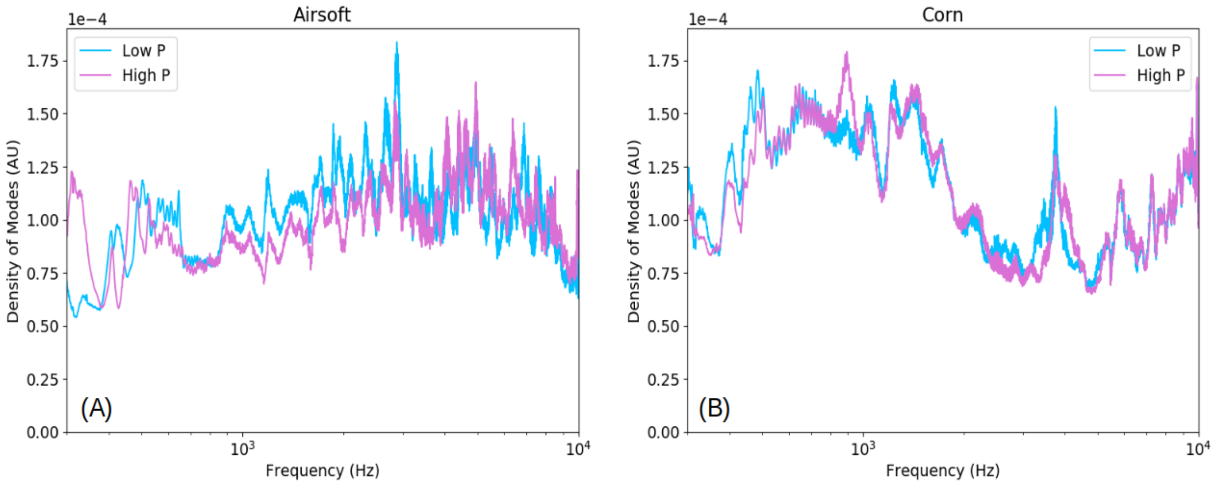

Figure 4: The density of modes for (A) Airsoft BBs and (B) corn.

4. Discussion

The main goal of this project is to measure the density of modes in both corn and Airsoft BB's. This difficult measurement requires carefully accounting for all possible sources of resonance and interference in order to measure features of the granular material itself and not just resonances and features of the experimental setup. Looking at the results in Figure 4, we can see that the two curves differ significantly from each other. The fact that there is a large difference between these two curves is strong evidence that the curves represent features of the granular system itself and not of the setup.

While we are confident that we are measuring features of the granular materials, these curves may differ from the actual density of modes due to the correction factor that was obtained in Figure 3 from the average spectrum of the piezoelectric sensors. While many of the prominent features in Figure 3 are likely strong resonances of the experimental system, it is possible that some of these features are granular features that are common to both the corn and the Airsoft BB's. Future studies that compare the procedure in this paper with an injection of sound containing a truly flat frequency spectrum similar to what was done in previous experiments[10] would be very interesting. If these two methods could be shown to produce similar results, the procedure described in this work would have the advantage of greater experimental simplicity and versatility in its application.

5. Conclusion

In conclusion, this experiment succeeded in being able to design and build an experimental setup for the purpose of measuring the granular density of modes. This work suggests a novel technique for measuring the granular density of modes in a real 3D material in order to access information about the jamming transition, and represents a step towards understanding granular materials and the jamming transition more fully.

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